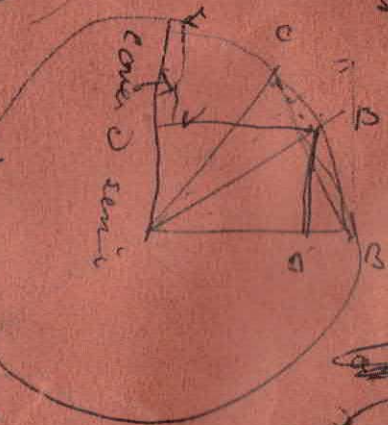


To find square of a decimal
 the number is made 100 times smaller
 i.e. 25 = .25 and the answer is
 made 10 times smaller = .00625
 (provided no. of even zeros and
 = i.e. using right side of rule.
 NB: = if odd zero use left side

64
 8
 173 3.20320
 173 55

~~It is the hypotenuse~~

= area by which
 where falls short of
 00 = covers
 same



175 odd to be

~~Amount of which the
 falls short of unit~~

P.I.C. SLIDE RULES

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{625}{256}$$

21 625
 572
 113
 625
 256
 2 112
 256

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STUDENT AND STANDARD
PATTERN

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INSTRUCTIONS FOR "P.I.C." SLIDE RULES

THE STANDARD PATTERN SLIDE RULE

1. Description—

The slide rule is an instrument used primarily for the multiplication and division of numbers, these being two of the fundamental operations of arithmetic. The operation is done mechanically by a process equivalent to logarithmic addition and subtraction. The numbers are read on logarithmic scales, but the logarithms themselves do not appear in the process.

The rule can also be applied to the calculation of powers and roots, logarithms and antilogarithms, the trigonometrical functions of angles, and generally to numerical problems in practical mathematics, applied mechanics, quantities, costing, structural and machine design, etc. It is indispensable in the technical college, the laboratory and the drawing office. It is in constant use by civil, mechanical and electrical engineers, and by many others and in many special cases too numerous to specify.

The rule consists of a body having a longitudinal groove in which a sliding strip can be moved, there being sufficient friction to prevent slipping. A glass cursor, moving under spring control, carries a fine transverse line (to be referred to as K), and this can be set in agreement with any graduation of the scales on the face of the body or the slide.

On the upper portions of the faces of the body and slide two identical and adjacent logarithmic scales, named A and B, are cut, the left and right halves of which are in duplicate, except that the former is figured from 1 to 10, and the latter from 10 to 100.

The lower pair of scales, C, D, on the slide and body, are again identical and adjacent; here the figuring extends only from 1 to 10. The unit of length used for setting out C and D is double that for A and B.

Other scales, marked S, T, and L, found on the under surface of the slide, will be described later. See Arts. 10 and 12.

2. Setting Out a Log Scale—

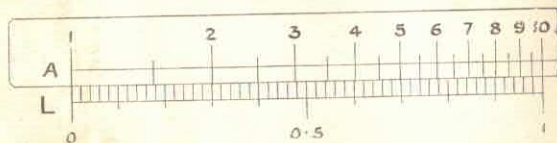


FIG. 1—PLOTING A LOG SCALE

In Fig. 1, the upper portion shows a part of the A scale of a 25 cm. slide rule, reduced in size and with the subdivisions omitted. The plain scale L below A is such as might be used in plotting the log scale on drawing paper, making use of a table of logarithms. It will be seen that numbers are read on the log scale and the mantissae of their logarithms on the plain scale. Conversely, the mantissa of a logarithm being read on the plain scale L, its anti-logarithm will be found on the log scale A.

See also Art. 12.

3. Reading the Scales—

For beginners, the first thing to do is to learn how to read the logarithmic scales.

In Fig. 2 several of the divisions of Fig. 1 are shown enlarged with the subdivisions put in.

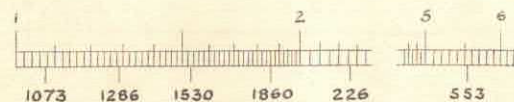


FIG. 2—READING A LOG SCALE

Slide rule scales are generally read to an accuracy of three significant figures, irrespectively of the position of the decimal point. (Often to four figures at the left end, where the first figure is unity). The first figure of the reading is got from the numbers stamped on the rule. The second is deduced by counting the subdivisions. The third (with the fourth if present) is an estimation, expressed decimally, of the fractional position of a point or line in the space of a subdivision. (Perhaps also with some counting.) For illustrations see the readings given in Fig. 2, and for practice use the rule itself.

Again, except in the extraction of roots (see Art. 11), the position of the decimal point when reading a number is ignored. Thus in Fig. 2, the reading 226, for instance, would serve for any one of the numbers

... 0.0226, ... 2.26, ... 22600, ... etc.

Moreover, with the reservation as to roots, any number may be read on either the left or the right hand half of scales A and B, regardless of the fact that these halves are figured 1 to 10 and 10 to 100 respectively.

Thus the range of a log scale (for positive numbers), like that of a table of logarithms, is unlimited.

4. Theory of the Rule—

Fig. 3 shows portions of the A and B scales of a slide rule, again with the subdivision lines omitted.

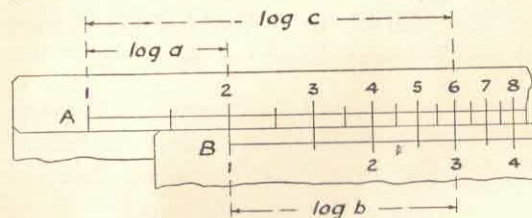


FIG. 3—MULTIPLICATION AND DIVISION

Let $c = ab$, then $\log c = \log a + \log b$

also $a = \frac{c}{b}$ and $\log a = \log c - \log b$

Thus to multiply or divide we in effect add or subtract logarithms. This is illustrated in Fig. 3, for the case in which $a = 2$, $b = 3$, and $c = 6$

Preliminary Exercises, using simple numbers.

Multiplication. (a) Find $2 \times 2.5 \times 3 \times 4 \times 6 \times \dots$

Set 1 on B against 2 on A; bring the cursor K over 2.5 on B. On A, under K, read ... Interim answer, 15.

Set 1 on B to K; set K to 3 on B ... Int. ans. 15.

Set 1 on B to K; set K to 4 on B ... Int. ans. 60.

Set 10 on B to K; set K to 6 on B ... Int. ans. 360.

And so on for any number of factors.

Division. (b) Calculate $\frac{150}{2 \times 2.5 \times 3 \times 5 \times \dots}$

Set K over 150 on A; bring 2 on B under K. On A, against 1 or 10 on B, read ... Interim answer, 75.

Set K to 10 on B; set 25 on B to K ... Int. ans. 30.

Set K to 10 on B; set 3 on B to K ... Int. ans. 10.

Set K to 1 on B; set 5 on B to K ... Int. ans. 2.

May be continued indefinitely.

The two combined. (c) Calculate $\frac{6 \times 3 \times 8 \times \dots}{2 \times 4.5 \times 5 \times \dots}$

Set K to 6 on A; set 2 on B to K; read ... Interim ans. 3.

Set K to 3 on B. Ans. 9. Set 4.5 on B to K. Ans. 2.

Set K to 8 on B. Ans. 16. Set 5 on B to K. Ans. 3.2.

etc.

In (a) we divide by unity between each multiplication; in (b) we multiply by unity between each division; in (c) the factors above and below the line are taken alternately, no unity terms being needed. In all cases the settings of cursor and slide alternate.

Preliminary Notes and Observations.

The following notes will repay careful study and confirmation.

The right and left halves of A and B provide an alternative choice of scale for general readings, and at the lines marked 1, 10, 100, a triple choice for unity readings.

The first multiplier and the answers are read on a body scale; all other readings are made on the slide. In practice the interim answers are not read.

In order to multiply we move the cursor; to divide we move the slide.

See Art. 6 for a more concise statement of rules.

Further Note. Observe that cursor readings on A (or B) are the squares of the corresponding readings on D (or C).

5. Continued Multiplication and Division—

While either pair of scales can be used for multiplication and division, or for combined or continued operations, it is to be noted that although C and D give greater accuracy, A and B are more convenient in general working. Thus if in finding $0.55 \times 2.8 = 1.54$, we set 1 on C to 55 on D, it is seen that the slide projects so far to the right that the result cannot be read on D under 28 on C. In such cases, K is brought to 1 on C and 10 on C to K, when 154 on D can be read under 28 on C. This traversing of the slide (which can be made at any stage in a calculation without affecting the final result) is avoided when working with the duplicated upper scales, and hence their use is advised except in cases where greater accuracy is desired.

Example 1. Find 54.7×0.0342 . *Ans.* 1.871

First, ignore the decimal points, and work as if the numbers were 547 and 342.

Set 1 on scale B against 547 on scale A.

Set the cursor line K over 342 on scale B.

Under K, on scale A, read 1871. This gives the answer except for the position of the decimal point.

To locate the latter, roughly estimate the answer, using round numbers, thus:

Approximate answer $= 55 \times \frac{1}{30} = 1.83$. This affords a rough check on the slide rule working; determines the position of the decimal point; and, along with the previous reading 1871, gives the complete answer 1.871, as written above.

Note. Abbreviation. In future, in order to save space and time, a statement such as "32 on scale A" may be shortened to "A 32."

Example 2. Find $27.38 \times 1.31 \times 0.332$. *Ans.* 11.91

Set B 1 to A 274; K to B 131; B 1 to K; K to B 332; under K read A 1191, which by correctly inserting the decimal point gives the answer written above.

Example 3 Find $\frac{7.4}{1.58}$ *Ans.* 4.68

In this simple case, as often happens in practical applications, an approximate answer (4.7) can be seen at once by inspection.

Two methods of working are given below. In method 1 we first multiply and then divide. In method 2 the steps are reversed. In each case two settings are required.

Method 1. Set K to A 74; bring B 158 under K; against B 1, 10 or 100 read, on A, the answer 4.68.

Method 2. Set B 158 against A 1 or 10; bring K to B 74; under K read, on A, the answer 4.68.

Example 4 Find $\frac{1}{64.7 \times 0.0254}$ *Ans.* 0.606

Set B 647 to A 10; K to B 10; B 254 to K;

Against B 1, 10 or 100 read A 606.

Locate the decimal point as in the answer above.

Example 5 Find $\frac{27.38 \times 1.31 \times 0.332}{64.7 \times 0.0254}$ *Ans.* 7.22

Set B 10 to A 274; K to B 131; B 647 to K; K to B 332; B 254 to K. Against B 1, 10 or 100 read A 722 and get the complete answer as given above.

Example 6 Find $\frac{2.17^2 \times 0.76}{7.14 \times 4.28^2}$ *Ans.* 0.0274

Set K to D 217; B 714 to K; K to B 76; C 428 to K; against B 10 read A 274, and then get 0.0274 for the complete answer on B as before.

Important Note. Observe that numbers, when they are to be squared, are to be read on one or other of the lower scales C or D.

6. Working Rules—

If the beginner will study the solutions of the examples just given, along with the theory of article 4, and work the *Exercises for practice* which follow, he will find that the operations for continued multiplication and division are standardised and simplified by being brought within the compass of the following four rules:

Rule 1. The first and last readings must be taken on a body scale, intermediate readings on the slide.

Rule 2. The cursor multiplies; the slide divides. These operations must take place alternately.

Rule 3. If the cursor be moved last, the answer is read under K. If the slide be moved last it is read against a unit line on the slide.

Rule 4. The answer being read on A, numbers to be squared must be read on C or D.

From the above it is seen that in continued multiplication or continued division, we must divide or multiply by unity between each of the given numbers as symbolized by the equations:

$$a \times b \times c \times \dots = \frac{a \times b \times c \times \dots}{1 \times 1 \times \dots},$$

$$\frac{1}{p \times q \times r \times \dots} = \frac{1 \times 1 \times 1 \times \dots}{p \times q \times r \times \dots}$$

whereas in a fraction such as

$$\frac{a \times b \times c \times \dots}{p \times q \times \dots}$$

the upper and lower factors can be taken alternately, and thus very few, if any, intermediate unity terms are needed. In fact, *none* is required when the number of terms above the line is *one less than*, *equal to*, or *one more than*, the number below.

Graduated exercises for practice in slide rule multiplication and division.

Verify the following calculations. In each case locate the decimal point and roughly check your answer after the manner previously shown in the working of Examples 1 and 3.

$$2 \times 5 = 10.$$

$$4.3 \times 2.5 = 10.75.$$

$$4.2 \times 26 \times 0.15 = 16.38.$$

$$4.2 \times 2.6^2 \times 1.5 = 42.6.$$

$$\frac{1}{8} = 0.125.$$

$$\frac{47}{61} = 0.771.$$

$$\frac{47^2}{61^2} = 0.594.$$

$$\frac{43 \times 36}{71} = 21.8.$$

$$\frac{3.14 \times 6.07 \times 21.6}{0.471} = 875.$$

$$2.8 \times 0.103^2 \times 51.5^2 \times 927 \times 0.0096^2 \times 37.2 = 250.$$

$$\frac{1}{7.3 \times 1.008 \times 4.03^2 \times 0.535 \times 89 \times 3.77^2} = 0.1236.$$

$$\frac{2.8^2 \times 0.103 \times 51.5 \times 927^2 \times 0.0096 \times 37.2^2}{7.3 \times 1.008^2 \times 0.403 \times 535^2 \times 89 \times 3.77} = 1.654.$$

$$4 \times 3 = 12.$$

$$4.35 \times 2.05 = 8.92.$$

$$3.14 \times 78 \times 86.1 = 21100.$$

$$3.14 \times 7.8^2 \times 8.61^2 = 14160.$$

$$\frac{9}{32} = 0.281.$$

$$\frac{47^2}{61} = 36.2.$$

$$\frac{47}{61^2} = 0.01263.$$

$$\frac{0.43}{36 \times 71} = 0.0001682.$$

$$\frac{21600}{3.14 \times 60.7 \times 47.1} = 2.41.$$

7. Gauge Marks or Constants—

The mark π , reading 314 on scales A and B is the well-known ratio of the circumference of a circle to its diameter (3.142).

The mark "M" also found on A and B reads 318; and $0.3183 = 1/\pi$, the reciprocal of π .

The marks "C" and "C₁" on scales C and D are, by using the cursor, both found to give the reading 1273 on B and A respectively. Now $1.273 = 4/\pi$, the reciprocal of $\pi/4$.

The use of these gauge marks may be illustrated thus:

Example 7. Find the curved surface and the volume of a cylinder of length l and diameter d .

$$\text{surface} = l \pi d = \frac{l d}{M}$$

$$\text{volume} = l \frac{\pi}{4} d^2 = \frac{l d^2}{C}$$

and in each case one setting of the cursor is saved by substituting the divisors M or $1/\pi$ and C or $4/\pi$ for the multipliers π and $\pi/4$.

For the same reason, physical constants are sometimes tabulated for use as divisors; thus specific volumes, $\text{ft}^3/\text{lb.}$, may be given instead of densities, lb./ft^3 .

Many useful constants and conversion factors will be found tabulated on the back of the rule.

Exercises.

1. Find the curved surface and volume of

i. A cylinder, diameter 2.25 ft., length, 3.62 ft.

Ans. 25.6 ft^2 , 14.40 ft^3

ii. A cone, diameter of base, 33 in., height, 39 in.

Ans. Surface = $\pi d \times \text{half slant height} = 16.38 \text{ ft}^2$

$$\text{Volume} = \frac{1}{3} \times \frac{\pi}{4} d^2 \times \text{height} = 6.43 \text{ ft}^3$$

2. The diameter of a spherical steel ball measures 2.73 in. Find its surface, volume, and weight.

Ans. Surface = $4\pi r^2 = 23.4 \text{ in}^2$;

$$\text{Volume} = \frac{4}{3} \pi r^3 = 10.65 \text{ in}^3. \text{ Weight} = 3.02 \text{ lb.}$$

8. Ratio, Proportion and the Rule of Three.

Geometrical Progression—

If two lines at a constant distance apart be moved over a log scale, the ratio p/q , of any pair of simultaneous

readings p and q , is constant. For, the distance between the lines is $\log p - \log q$, that is $\log p/q$, and since the distance is constant so is the ratio p/q .

For the same reason if a pair of similar log scales like A and B, or C and D be set in any relative position, the ratio of any pair of cursor readings is constant. This property enables us to apply arithmetically the rule of three or proportion. See Fig. 3, Art. 4.

It also follows that any set of equidistant readings on a log scale form a geometrical progression. If now the slide be inverted so that the plain scale L is brought to the top, this scale will become available for the setting of the cursor in any required series of equidistant positions.

For a more general statement of some proportional properties of the rule, let x, y, z be a set of cursor readings on the scales A, B, C, the slide being set in any stationary position, then for all such readings x is proportional to y and to the square of z . Or in symbols

$$\frac{x}{y} = \frac{x_1}{y_1} = \dots = \text{constant} = \frac{a}{1} = \frac{1}{b}.$$

$$\frac{x}{z^2} = \frac{x_1}{z_1^2} = \dots = \text{constant} = \frac{a}{1} = \frac{1}{c^2}.$$

where a and b are the readings on A and B against end lines on B and A and a and c are the readings on A and C against end lines on C and A.

In illustration, if linear dimensions z of similar figures be read on C, proportional areas x will be found on A.

Next, with the slide reversed in direction, let x, y, z denote sets of scale readings as before, then x is inversely proportional to y and to z^2 . That is—

$$x y = x_1 y_1 = \dots = \text{constant} = a \times 1 = 1 \times b.$$

$$x z^2 = x_1 z_1^2 = \dots = \text{constant} = a \times 1 = 1 \times c^2.$$

If $x = z$, we get third and two-thirds powers and roots. Another application of these proportional properties is found in reading off the co-ordinates of points on a straight line, parabola, hyperbola or cubic xz^2 constant.

Example 8. A joint of meat costing 5s. 2d. is found to weigh 3lb. $4\frac{1}{2}$ oz.; what is the price per lb.? *Ans.* 1/7.

$$1\text{lb.} = 16\text{oz.}; 3\text{lb. } 4\frac{1}{2}\text{oz.} = 52\cdot5\text{oz.}; 5\text{s. } 2\text{d.} = 62\text{d.}$$

By aid of the cursor set B 525 against A 62, and against B 16 read A 190 or 1s. 7d.

Example 9. An alloy contains 52 parts of copper, 27 of tin and 7 of zinc; find its percentage composition.

$$\text{Ans. } 60\cdot5, 31\cdot4, 8\cdot1$$

$$\text{Adding, we get } 52 + 27 + 7 = 86$$

$$\text{And we require } x + y + z = 100$$

Set 86 on C against 10 on D and against 52, 27 and 7 on C read x , y and z on D.

Note. The lower scales are used in this case because special accuracy is desirable.

Example 10. The numbers of teeth in a set of change wheels for a lathe range from 15 to 120, in steps of 5. Find whether a pair of wheels can be selected from the set which will give approximately a velocity ratio of

$$(a), \pi; \quad (b), \frac{1 \text{ cm.}}{1 \text{ in.}}.$$

(a) Set B π to A 10. Inspect other coinciding pairs of scale readings on B and A and select B 110 and A 35 as giving a suitable choice of wheels.

$$(b) \quad \frac{1 \text{ cm.}}{1 \text{ in.}} = \frac{0\cdot393708}{1\cdot0} = \frac{6\cdot29933}{16} = \frac{63}{160} \text{ nearly.}$$

No pair of wheels in the set having numbers of teeth near this ratio can be discovered.

$$\text{But since } \frac{63}{160} = \frac{3 \times 21}{8 \times 20} = \frac{30 \times 105}{80 \times 100},$$

it is seen to be possible, with a train of four wheels selected from the set, to cut a metric screw with quite remarkable accuracy, using a guide screw having a pitch based on the inch. The pitch error is in fact only 1 in 9370.

Exercises.

1. Solve the following four equations, which are expressed in proportional form.

$$\frac{x}{6} = \frac{11}{29} \quad \text{Ans. } 2\cdot28, \quad \frac{4\cdot3}{x} = \frac{27\cdot3}{0\cdot314} \quad \text{Ans. } 0\cdot0495.$$

$$\frac{x}{2\cdot81^2} = \frac{14\cdot1}{3\cdot72^2} \quad \text{Ans. } 805, \quad \frac{3\cdot45^2}{17\cdot2} = \frac{37}{x} \quad \text{Ans. } 53\cdot5.$$

2. From the set of change wheels specified in Ex. 10 above, select the pair of wheels which will give a velocity ratio nearest to the ratio of half a lunar month to a week. The mean lunar month is equal to 29·53 days.

$$\text{Ans. } 95 \text{ and } 45 \text{ teeth.}$$

3. An alloy consists of copper, tin, and lead in the proportions 17, 29, and 42. A new alloy is made by adding 2 cwt. of copper to each ton of the old alloy. Find the percentage compositions of the alloys.

$$\text{Ans. } 19\cdot3, 33\cdot0, 47\cdot7. \quad 26\cdot6, 30\cdot0, 43\cdot4.$$

4. A geometrical progression has seven terms, the first being 16·4 and the sixth 92·2. Determine the progression and its common ratio r .

$$\text{Ans. } 16\cdot4, 23\cdot2, 32\cdot7, 46\cdot2, 65\cdot3, 92\cdot2, 130\cdot2. \quad r = 1\cdot413.$$

9. Reciprocals. Hyperbolic Expansion Curves—

(a) If two similar log scales A and B (or C and D) be set in any relative position, the upward and downward readings on A and B, against end or unit lines on B and A are reciprocals, and their product is unity.

(b) If the slide be removed from the body, reversed in direction and then reinserted, so that the figures read upside down and from right to left, the two scales B and C become reciprocal scales of A and D respectively. The following are two of their useful properties:

i. Set the reversed slide in its central position in the body. Then pairs of cursor readings on A and B (also on C and D) are reciprocals.

ii. Displace the reversed slide to any other position. The upward and downward cursor readings against end lines, as in (a), are now equal to one another. And the product of any pair of simultaneous cursor readings is constant and equal to the end reading.

This property is useful in calculations relating to hyperbolic expansion curves, $p v = \text{a constant}$.

Exercises.

1. Find the reciprocals of π , $\pi/4$, e and $\log e$.

Ans. 0.3183, 1.273, 0.3680, 2.303.

2. An expansion curve from the indicator diagram of a steam engine follows the law $p v = 378$. Find the pressures corresponding to volumes of 2.1, 4.68, 7.53, and the volumes at pressures of 233, 147, 76.1.

Ans. 180.0, 80.8, 50.2 and 1.622, 2.57, 4.98.

* 10. Trigonometrical Ratios—

On the under face of the slide will be found two scales marked S and T; these are logarithmic scales of

* See footnote, page 31.

sines and tangents of angles,* the angles being figured in degrees and minutes. In the two slots at the ends of the body there are reference lines or marks.

Scale S reads from 35 minutes to 90 degrees; it is used in conjunction with scale A. Scale T reads from about 5.7° to 45° , and it is employed along with D. Natural sines and tangents are read off on A and D in the manner described below.

NOTE.—The subsequent instructions relate to the forementioned combinations of scales. Other combinations may be encountered, in which case the reader is expected to infer the slight modifications of text applicable.

Since, for any angle, the sine and cosecant, cosine and secant, tangent and cotangent are in each case reciprocal; and since further, for complementary angles θ and $90 - \theta$, the sine and cosine, tangent and cotangent, secant and cosecant are respectively equal, it is found possible to read off the six trigonometrical ratios for all angles which do not fall below the range of the scales S and T.

To express an angle in *radians* we divide degrees by 57.30, this number being equal to $180/\pi$. For small angles less than 6° (which thus fall below the range of T), the sine, tangent and radian measures, expressed to an accuracy of one per cent., are all equal. Hence the tangent of an angle less than 6° can be calculated from the radian rule just given. Alternatively, both tangents and radians, as well as sines, of small angles can be read on A in connection with S, since the latter extends downwards to angles less than one degree.

Example 11. Find the six trigonometrical ratios for an angle of 33.3° .

Set S 33.3 against the mark in one of the end slots. Then on scales B and A, against the end lines, take

* See footnote, page 31.

downward and upward readings,* as if for reciprocals. These readings, with the decimal points inserted give the answers:

$$\sin 33.3^\circ = 0.549; \quad \operatorname{cosec} 33.3^\circ = 1.82$$

Repeat this process for the complementary angle $90^\circ - 33.3^\circ$ or 56.7° . We thus get:

$$\cos 33.3^\circ = \sin 56.7^\circ = 0.835; \quad \sec 33.3^\circ = 1.198$$

Lastly, set T 33.3° against the mark in the left hand slot. On scales C and D, against the end lines, take the upward and downward readings, which, with decimal points added give

$$\tan 33.3^\circ = 0.656 \quad \cot 33.3^\circ = 1.525$$

Example 12. A force of 27 pounds acts along a line which makes $41\frac{1}{2}^\circ$ with the horizontal. Find its horizontal and vertical components. *Ans.* 20.2 pds.; 17.88 pds.

We require to find $27 \cos 41\frac{1}{2}^\circ$ and $27 \sin 41\frac{1}{2}^\circ$. That is $27 \sin 48\frac{1}{2}^\circ$ and $27 \sin 41\frac{1}{2}^\circ$. Invert the slide so that scale S now lies alongside its companion A.

Set the right hand end line of S against A 27. Take cursor readings on A against the readings $48\frac{1}{2}$ and $41\frac{1}{2}$ on S. These with the decimal points give the answers

$$27 \sin 48\frac{1}{2}^\circ = 20.2 \quad 27 \sin 41\frac{1}{2}^\circ = 17.88.$$

Example 13. Given the three angles α, β, γ of a triangle, find numbers a, b, c which are proportional to its sides.

$$\text{We have, for any triangle, } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Set the inverted slide in any position, and against α, β, γ on S read a, b, c on A.

* See footnote, page 31.

* *Example 14.* Given the sides a, b, c of a triangle, to find the angles α, β, γ .

Against a, b, c on A take readings α, β, γ on S. Then adjust the slide, until by trial, the sum $\alpha + \beta + \gamma$ of the readings is 180° .

Example 15. The horizontal and vertical components of a vector are 3.27 and 2.46 units. Find the magnitude v of the vector and the angle δ which its line makes with the horizontal. *Ans.* 4.09, $36^\circ 57'$.

$$\tan \delta = \frac{2.46}{3.27} = 0.752 \text{ (by slide rule division)}$$

Bring the inverted slide to its middle position in the body. Against 752 on D read, on T, the required angle $\delta = 36^\circ 57'$.

For the magnitude v we have

$$v = \frac{\text{hor. comp.}}{\cos \delta} = \frac{3.27}{\sin (90 - \delta)} = \frac{3.27}{\sin 53^\circ 3'}$$

Set $53^\circ 3'$ on S against 3.27 on A. Against the end line on S read, on A, the answer $v = 4.09$ units.

Exercises.

1. Find the sine, cosine, and tangent of an angle of 128.5° .

$$\text{Ans. } 0.783, -0.623, 1.257.$$

2. Find the six trigonometrical ratios of 1 radian or 57.3° .

$$\text{Ans. } \sin 1 \text{ radn.} = 0.841, \cos 1 \text{ radn.} = 0.540, \tan 1 \text{ radn.} = 1.558, \\ \operatorname{cosec} 1 \text{ radn.} = 1.188, \sec 1 \text{ radn.} = 1.851, \cot 1 \text{ radn.} = 0.642.$$

3. Reduce $10^\circ, 65^\circ, 180^\circ$, to radians, and $0.15\pi, 0.76, 2.2$ radians to degrees.

$$\text{Ans. } 0.1745, 1.134, 3.142; 27^\circ, 43.5^\circ, 126.1^\circ.$$

* See footnote, page 31.

4. Find the sine, tangent, and radian measure of 4.5° .

Ans. 0.0785, 0.0787, 0.0785.

5. Solve the following three equations.

$$\frac{x}{\sin 14.2^\circ} = \frac{3.78}{\sin 54.6^\circ} \quad \text{Ans. } x = 1.14.$$

$$\frac{20.6x}{\tan^2 14.2^\circ} = \frac{3.78}{\sin 54.6^\circ} \quad \text{Ans. } x = 0.01442.$$

$$\frac{x \sin 14.2^\circ}{20.6} = \frac{3.78}{\sin 54.6^\circ} \quad \text{Ans. } x = 389.$$

6. If $a \sin \theta + b \cos \theta = r \sin (\theta + \alpha)$, then $\alpha = \tan^{-1} \frac{b}{a}$ and $r = \frac{b}{\sin \alpha}$. Find α and r when $a = 3.26$, $b = 2.16$.

Ans. $\alpha = 33.5^\circ$, $r = 3.91$.

7. Given $y = 7.3 \sin 3x - x \tan 2x$, find y_1 and y_2 corresponding to $x_1 = 0.681$ rad. (39°) and $x_2 = 0.698$ rad. (40°). Hence by linear interpolation deduce an approximate root of the equation $7.3 \sin 3x = x \tan 2x$.

Ans. 0.11, -0.31, 0.685.

11. Powers and Roots—

Squares.

Example 16. Given a , find a^2 .

Set the cursor to a on D and read the answer on A.

Cubes.

Example 17. Given a , find a^3 . That is find $a^2 \times a$.

Set C1 to D a ; K to B a ; against K read, on A, the answer a^3 .

Square roots. In extracting roots the position of the decimal point has first to be considered. This is readily

done by "pointing," as in ordinary arithmetic. See the following examples.

Example 18. Find the square root of 2370. *Ans.* 48.7

In pointing, the digits are grouped in pairs beginning at the unit figure; we thus get 2370. By inspection, the required root is roughly 49. Now square 49, as in Example 16, and thus bring the cursor near to the desired position. Then move K slightly so as to read exactly 237 on A. The cursor reading on D now gives the answer 48.7.

Example 19. Find the square root of 0.0237. *Ans.* 0.154.

Pointing gives 0.0237, and inspection gives 0.15 as the first approximation to the root. Then operating as in Example 18 we find the answer to be 0.154.

Cube roots.

Example 20. Find the cube root of 2370. *Ans.* 13.33.

Pointing, in groups of three, gives 2370. Inspection gives 13 as the first guess for the cube root. The cubing of 13, after the manner of Example 17, brings the cursor near to the desired position.

Now shift the cursor slightly so as to read 237 on A, and then adjust the slide until the two readings on D and B are identical. Either of these gives the answer, which is 13.33.

Example 21. Find the cube root of 0.0237. *Ans.* 0.2873.

Pointing gives 0.0237, inspection gives for the answer 0.28 nearly. Operating, as in Example 20, with this approximate root, we get 0.2873 for the final answer.

The procedure here given for the extraction of roots is perhaps somewhat unusual, but it is reliable, easily remembered and readily recalled.

Exercises.

1. Find a^2, a^3, a^4, a^5, a^6 , when $a = 1.48$.

Ans. 2.19, 3.24, 4.80, 7.10, 10.5.

2. Find the square, cube, and fourth roots of (a) 3100, (b) 0.031.

Ans. (a) 55.7, 14.58, 7.46; (b) 0.1761, 0.3142, 0.420.

3. Find the cube root of $\frac{25.7 \times 33000}{42.6 \times 183 \times \pi}$.

Ans. 3.26.

4. Verify that $x = 0.315$ is a root of the equation $1000x^3 - 40x^2 = 4$.

5. Solve the equation $\frac{2.22x}{\sqrt[3]{\tan^2 31^\circ}} = \frac{43}{\sqrt[3]{\sin^2 58^\circ}}$

Ans. $x = 15.39$.

12. Logarithms—

The plain scale L, found on the face of the stock or the reverse side of the slide, and the log scale D on the front face of the body are used in conjunction with each other for the reading of logarithms and antilogarithms or numbers. A reference to Art. 2, will serve as an explanation. The mantissa only of the logarithm is read on L. The characteristic is determined in the usual manner by the position of the decimal point, and *vice versa*. Scale L may be employed either with or without an inversion of the slide, after the manner previously explained for scales S and T.

Example 22. Calculate $0.0465^{-1.42}$ Ans. 78.2.

First, find $\log 0.0465$, thus: Against 465 on D read the mantissa 0.667 on L. (Either by an inversion of the

slide or by use of the mark in the right hand slot). Add the characteristic -2 and thus obtain

$$\log 0.0465 = \bar{2}.667 = -2 + 0.667 = -1.333$$

$$\text{Now find } -1.333 \times (-1.42) = 1.893$$

This is the logarithm of the answer. To find its antilogarithm:

Against the mantissa 893 on L read 782 on D. Insert the decimal point as indicated by the characteristic 1. We thus obtain, for the final answer, 78.2.

Note. Hyperbolic logarithms are found by multiplying ordinary logarithms by 2.303 or dividing by 0.4343. That is

$$\log_e = 2.303 \log_{10} \text{ or } \frac{\log_{10}}{0.4343}$$

Exercises.

1. Obtain $\log_{10}$ and $\log_e$ of the following numbers:

50	10	2.718	0	0.2	0.1
Ans. 1.699	1.0	0.4343	0	1.301	1.0
3.91	2.303	1.0	0	-1.610	-2.303

2. Given the following logs find their antilogs:

1.8	0.4	0.2	0.1	1.7	1.5	1.3
Ans. 63.1	2.512	1.585	1.259	0.501	0.316	0.1995

3. Find $32^{1.4}$ $3.2^{1.4}$ $0.32^{1.4}$ $6.31^{2.5}$ $0.631^{-2.5}$ 0.85^{-15}

Ans. 128 5.09 0.203 100 0.316 11.46

4. The expansion curve of the indicator diagram of a gas engine has the equation $pv^{1.3} = 222$. If $p = 100$, find v . If $v = 2$, find p .

Ans. 1.847, 90.4.

13. Locating the Decimal Point—

A method of locating the decimal point in an answer, available as an alternative to that hitherto employed is now given, though it is seldom used in practice. It applies only to calculations made entirely on scales C and D. The method is based on the counting of digits in the numbers comprising the data, and on noting the position of the slide in the body after each step of the calculation. The complete process will now be described in connection with the following example:

$$\frac{30\cdot3 \times 4\cdot1 \times 0\cdot22 \times 0\cdot0065}{7\cdot12^2 \times 0\cdot000055} = 63\cdot7.$$

$\begin{matrix} (2) & (1) & (0) & (-2) \\ 30\cdot3 & \times & 4\cdot1 & \times & 0\cdot22 & \times & 0\cdot0065 \\ \hline & & 7\cdot12^2 & \times & 0\cdot000055 \\ & & (2) & & (-4) \end{matrix}$

The bracketed numbers give digits, counting from the decimal point. It will be seen that for numbers less than unity, we count the *cyphers* to the right of the point, and prefix the negative sign.

Let n denote the excess of the sum of the digits in the numerator over the sum for the denominator. That is

$$n = (2 + 1 + 0 - 2) - (2 - 4) = 1 - (-2) = 1 + 2 = 3.$$

Next, the signs "Quot., + 1" and "Prod., - 1" found on the body, mean that in making a calculation, whenever the slide is found projecting to the *right*, immediately *after* a *division* or *multiplication* (i.e., by *slide* or *cursor* respectively), we must *add* or *subtract* 1 each time. Settings for division or multiplication by 1 are excluded from the reckoning.

In applying these rules to the above example it will be found that the minus steps exceed the plus steps by one.

The number of digits in the answer is therefore

$$n - 1 = 3 - 1 = 2$$

and the position of the decimal point is determined.

Exercises.

Verify the above rules in the following simple cases, taking the factors all separately, as written.

$$\begin{array}{lll} 2 \times 2 = 4. & 2 \times 2 \times 2 = 8. & 2 \times 2 \times 2 \times 2 = 16. \\ 5 \times 5 = 25. & 5 \times 5 \times 5 = 125. & 5 \times 5 \times 5 \times 5 = 625. \\ \frac{6}{3} = 2. & \frac{3}{6} = 0\cdot5. & \frac{50}{0\cdot002} = 25000. \\ \frac{1\cdot001}{2} = 0\cdot5. & \frac{0\cdot999}{2} = 0\cdot5. & \frac{0\cdot002}{50} = 0\cdot00004. \\ \frac{1}{2 \times 2} = 0\cdot25. & \frac{1}{2 \times 2 \times 2} = 0\cdot125. & \frac{1}{2 \times 2 \times 2 \times 2} = 0\cdot0625. \\ \frac{1}{4} = 0\cdot25. & \frac{1}{4 \times 4} = 0\cdot0625. & \frac{1}{4 \times 4 \times 4} = 0\cdot0156. \end{array}$$

The graduated exercises of Art. 6 are available for further practice.

* 14. Temperature-Resistance Scale for a Copper Circuit—

This scale, marked C°, is superposed on a portion of the tangent scale T, found on the under surface of the slide. It reads from 0 to 120 degrees centigrade, and gives, in conjunction with scale D, the varying temperatures of a copper circuit, corresponding to observed changing resistances of the circuit or *vice versa*, temperatures being read on C° and resistances on D.

The scale is due to Prof. Miles Walker, of the College of Technology, Manchester.

* See footnote, page 31.

Example 1. At a temperature of 10°C the resistance of the field-windings of a dynamo machine measures 42 ohms. Find the mean temperature of the circuit when the observed resistance is 47 ohms. *Ans.* 39.4°C .

Working. Invert the slide. Set 10° on C° against 42 on D; against 47 on D read the answer 39.4° on C° .

Example 2. At 15°C the resistance of a copper circuit is 42.8 ohms. Find the increase in the resistance corresponding to a rise of temperature of 30°C . *Ans.* 5.1 ohms.

Working. Set 15 on C° against 42.8 on D; against $(15 + 30)$ or 45° on C° read 47.9 on D. The increase in the resistance is thus $(47.9 - 42.8)$ or 5.1 ohms.

15. General Remarks—

As soon as the beginner, by sufficient practice, has become proficient in the fundamental operations of continued multiplication and division, he may, with advantage to himself, begin to experiment with the rule and to explore its possibilities. He will soon discover properties of the instrument not enumerated here, and new ways of working, some of which will no doubt be specially suited to his particular needs. Thus by observing and postulating the obvious proportional properties of the A and B scales, and the equally obvious relations of numbers and their squares between cursor readings on the lower and upper scales, it is easy for a beginner to establish for himself the four working rules of Art. 6, without his knowing anything whatever of logarithms.

16. Miscellaneous Examples for Practice and Illustration.

1. Using data found on the rule, verify to three figures the following conversion factors:

$$\begin{aligned} \text{gramme} &= 15.432 \text{ grains.} & \text{lb.} &= 0.4536 \text{ kgram.} \\ \text{km.} &= 0.621 \text{ mile.} & \text{yd}^3 &= 0.767 \text{ m}^3. \\ \text{ft. lb./min.}^2 &= 0.00384 \text{ cm. kgram./sec.}^2. \\ \text{kgram. cm.}^2/\text{sec.}^2 &= 1.526 \text{ ton yd.}^2/\text{hour}^2. \end{aligned}$$

2. Electric circuit. The following equations refer to three resistances in parallel and three condensers in series.

$$\frac{1}{R} = \frac{1}{4.76} + \frac{1}{8.23} + \frac{1}{11.5}, \quad \frac{1}{K} = \frac{1}{2.3} + \frac{1}{4.7} + \frac{1}{6.6}$$

Find R and K .

Ans. 2.39; 1.251.

3. The sides of a triangular plot of ground measure 123, 185, and 216 yards. Use the formula $\sqrt{s(s-a)(s-b)(s-c)}$ to calculate its area in square yards. Reduce this to acres.

Ans. 11,360, 2.35.

4. Masonry dam. Length 123 ft., depth 36 ft., section trapezoidal, width 11.5 and 21 ft. top and bottom, density 146 lb./ft.³. Find the volume and weight of the masonry in cubic yards and tons.

Ans. 2665, 4690.

5. Timber log. Length 18.17 ft., mean diameter 22.6 in., weight 1.06 tons, price 2s. 2½d. per cubic foot. Find the density of the log and its cost.

Ans. 46.9 lb./ft.³. £5 11s. 9d.

6. Four cylinder petrol engine. Cylinder capacity 1111 cm.³, speed 3850 revs./min., I.H.P. 20. Find p_e as would be measured from an indicator diagram.

Ans. 30.4 pd./in.².

Check the following numerical work:

$$1111 \text{ cm.}^3 = 1111 \text{ cm.}^3 \times (0.0328 \text{ ft./cm.})^3 = 0.3915 \text{ ft.}^3 = v \text{ say,}$$

$$144 p_e v \times 3850 = 20 \times 33000; p_e = \frac{660,000}{144 \times 3850 \times 0.3915} = 30.4.$$

7. A $4\frac{1}{4}$ inch steel shaft transmits 88 H.P. at 115 revs./min. Transverse modulus $G = 4700$ tons/in.². Find the torque T pd. ft., the stress q tons/in.², and the angular deflection θ radn./ft.

Ans. 4020, 1.428, 0.001712.

Check the following numerical work:

$$T = \frac{HP \times 33000}{2 \pi N} = \frac{88 \times 33000}{2 \pi \times 115} = 4020 \text{ pd. ft.}$$

$$q = \frac{16 T}{\pi d^3} = \frac{16 \times 4020 \times 12}{\pi \times (4.25)^3 \times 2240} = 1.428 \text{ ton/in.}^2$$

$$\theta = \frac{l q}{r G} = \frac{12 \times 1.428}{2.125 \times 4700} = 0.001712 \text{ radn./ft.}$$

8. Standard steel joist, $d = 9$ in., $I = 230$ in.⁴, $E = 13,500$ tons/in.², $f = 7.5$ tons/in.², span l ft., spread load W tons, deflection y ins.

(a) If $l = 14.5$, find W and y . (b) If $y = \frac{l}{360}$, find W and l .

Check the following slide rule work:

$$(a) W = \frac{16 f I}{l d} = \frac{16 \times 7.5 \times 230}{14.5 \times 12 \times 9} = 17.63 \text{ tons.}$$

$$y = \frac{5 W l^3}{384 E I} = \frac{5 \times 17.63 \times (14.5 \times 12)^3}{384 \times 13500 \times 230} = 0.389 \text{ in.}$$

$$(b) W = \frac{16 f I}{l d}, \quad \frac{l}{360} = \frac{5 W l^3}{384 E I}, \text{ from which}$$

$$W l = \frac{16 \times 7.5 \times 230}{9} = 3070; \quad W l^2 = \frac{384 E I}{5 \times 360} = 661,000, \text{ and}$$

$$l = \frac{661000}{3070 \times 12} = 18.0 \text{ ft.} \quad W = \frac{3070}{18 \times 12} = 14.2 \text{ tons.}$$

9. Solve the coplanar vector equation:

$$r_{\theta} = 4.21_{32^\circ} + 6.18_{51^\circ}.$$

Ans. $10.26_{43.3^\circ}$.

Method. Work by 0° and 90° components. Thus

$$r \cos \theta = 4.21 \cos 32^\circ + 6.18 \cos 51^\circ = a \text{ say.}$$

$$r \sin \theta = 4.21 \sin 32^\circ + 6.18 \sin 51^\circ = b \text{ say.}$$

$$\theta = \tan^{-1} \frac{b}{a}; \quad r = \frac{b}{\sin \theta}$$

10. The magnitudes and positions of three point masses are tabulated. Find their centre of mass.

No.	m	x	y	z	mx	my	mz
1	2.14	1.81	3.7	4.2			
2	3.47	-2.1	6.5	1.7			
3	5.26	7.3	4.3	2.6			
Sum		$\bar{x}$	$\bar{y}$	$\bar{z}$			

$$\text{Ans. } \bar{x} = 3.22; \quad \bar{y} = 4.88; \quad \bar{z} = 2.62.$$

Method. Use the formula $\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$, etc., and tabulate your slide rule readings as suggested.

11. Data as in Ex. 10. Find the moments of inertia of the system about the axes.

$$\text{Ans. } I_x = \sum m (y^2 + z^2) = 356; \quad I_y = 386; \quad I_z = 576.$$

Note. Tabulate your slide rule readings in columns headed x^2 , y^2 , z^2 , and $m (y^2 + z^2)$, $m (z^2 + x^2)$, $m (x^2 + y^2)$.

12. Two hinged arched rib. Load W at crown; horizontal thrust H , Tabular data measured from scale drawing.

No.	1	2	3	4	5
x	4.5	17.0	32.6	49.6	67.1
y	7.9	20.9	30.0	35.8	38.0
$x y$					
y^2					

Determine $\frac{H}{W}$, using the formula $2 \frac{H}{W} = \frac{x_1 y_1 + x_2 y_2 + \dots}{y_1^2 + y_2^2 + \dots}$

Ans. 0.691.

* NOTE: The instructions marked thus (*) apply only to the ordinary Standard Pattern Rules which include Orthodox Sine and Tangent Scales in their complement.

For the P.I.C. New Series Improved Standard Types (which incorporate patented Direct and Inverse Differential Scales for Trigonometrical computations) a separate booklet is issued, which fully explains the uses of these special scales and deals with their design.

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