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how to use THE **N 16 ELECTROLOG** SLIDE RULE


PICKETT
THE WORLD'S MOST ACCURATE
SLIDE RULES

PICKETT, INC. • PICKETT SQUARE • SANTA BARBARA, CALIFORNIA 93102


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FOREWORD

The need for a specialized electronic slide rule has been apparent for some time, and requested by electrical and electronic engineers. In circuit design and network analysis, the engineer is usually faced with the problems of solution of a number of basic relationships. At best, these are time consuming, particularly when confronted with a lengthy and complicated solution. This need, and the convenience of having a device capable of giving answers to preliminary design questions, both quickly and accurately, has lead to the development of the Pickett N-16 Electronic Slide Rule.

Here, in this Pickett all-metal slide rule, are embodied all of the advancements of the slide rule manufacturers' art; a completely new arrangement of scales to solve electrical and electronic problems, a special arrangement of conventional scales for electrical engineers, scale and graduation accuracy found in no other method of construction, and Pickett's exclusive eye-saver color.

The N-16 Electronic Slide Rule has been designed to solve the leading and lagging behavior of both RC and RL systems, LC resonant circuits and ideal transmission lines. An additional feature not before attempted in such a rule is a method of finding the decimal point for electronic calculations, which frequently spread over a range of more than 17 decades.

The side of the rule devoted to general purpose scales is also arranged with the electronic and electrical engineer in mind. The combined solution of hyperbolic and circular functions so prevalent in distributed systems are more readily handled than on conventional rules. The solution of right angle triangles required in changing from polar to rectangular coordinates is of special arrangement also, so that a solution of the phase angles from 0° to 90° is handled in one setting and without a change in procedure.

In the N-16 Instruction Manual, we have attempted to explain the function of this unusual rule, in a manner as simple and concise as the subject would allow. We are confident that a short period of study of the Manual and the Electronic Slide Rule will pay dividends in time and convenience.

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by π when transferred directly above with the indicator to CF. This also holds for the D and DF scales. Transferring from the DF and CF to the D and C divides by π .

At the top of the rule there is a set of Hyperbolic scales. The values of these functions are read on the D scale. In the middle of the slide are scales for circular or trigonometric functions. These scales have been calibrated in the reverse direction from the conventional mode. Such a reverse in direction makes it more convenient to convert from rectangular to polar coordinates which is of particular importance in solving electrical networks. The actual conversion method will be explained later. A CI scale is placed below the T scale so that the values of the trig functions could be read if desired. This, however, is seldom necessary in routine calculations. The Log Log scales are calibrated with the base e at the index. This base is more convenient in electrical work than base 10 for solution of functions in the time domain. Unfortunately, there is not enough room on the rule to include the reciprocal values of the Log Log functions which would have directly given functions of e^{-x} . It was felt, however, that of the two functions, e^x had greater total merit. Two additional scales are given. These are L ($\log_{10}$) and Ln ($\log_e$). Although these values can be found from the Log Log scales, there is enough need in electrical work for these functions to make their inclusion helpful.

Several special reference marks are included. Starting at the left index of the C or D scale, the first mark is just to the right of 1.4 and is the $\sqrt{2}$ value or approximately 1.414. The next mark, to the right of 57, is the number of degrees in one radian, approximately 57.29. The next mark is just to the right of 7.05 and is the value of $1/\sqrt{2} = \cos 45^\circ$ or approximately 0.707. The last mark of this set is just to the right of 7.85 and is the value of $\pi/4$ or approximately 0.785. These same reference marks are repeated on the CF and DF scales.

The Dual Base Log Log manual that is included with Model N-16 discusses the use of the Math Scales on this rule. The section on hyperbolic functions is based on the scales found on the slide instead of the body of the rule. The necessary conversion of technique will be covered in this manual. The use of the L and Ln scales is covered in a separate booklet, also included with the rule.

1.00 SLIDE RULE DESCRIPTION

The Model N-16 ELECTRONIC slide rule was designed to aid Electrical and Electronic Engineers. The purpose was to combine two different sets of scales in one rule. One set fulfills conventional slide rule needs, while a second set of special scales will solve with ease the routine calculations that recur in circuit analysis. The conventional set will be referred to as the Math scales, and the special set as the Electronic scales.

Fig. 1a (front side) shows the Math scales of the rule. The main scales are the C and D which are located at the bottom interface of the slide and the body of the rule respectively. These scales have one cycle for the length of the rule. At the upper interface are the CF and DF scales. These are the same as the C and D scale except that they start at π . This allows many readings to be made without resetting the rule since any opposed portion of these two combinations will read the same. Many calculations involving π can be made in a single setting since any number on C scale is multiplied by π when transferred directly above with the indicator to CF. This also holds for the D and DF scales. Transferring from the DF and CF to the D and C divides by π .

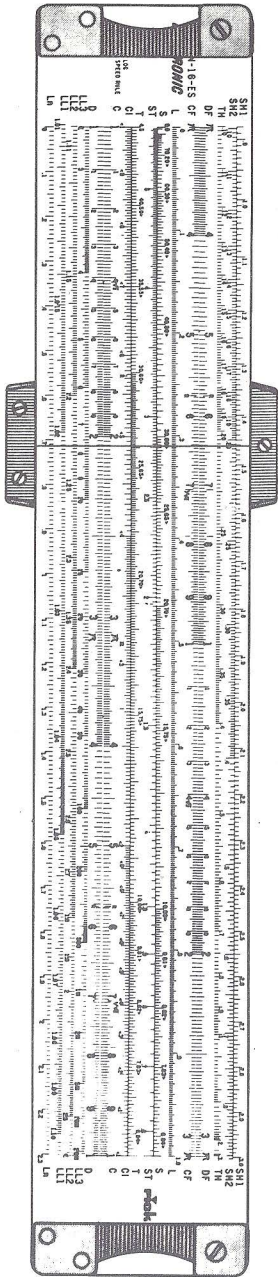


Figure 1a

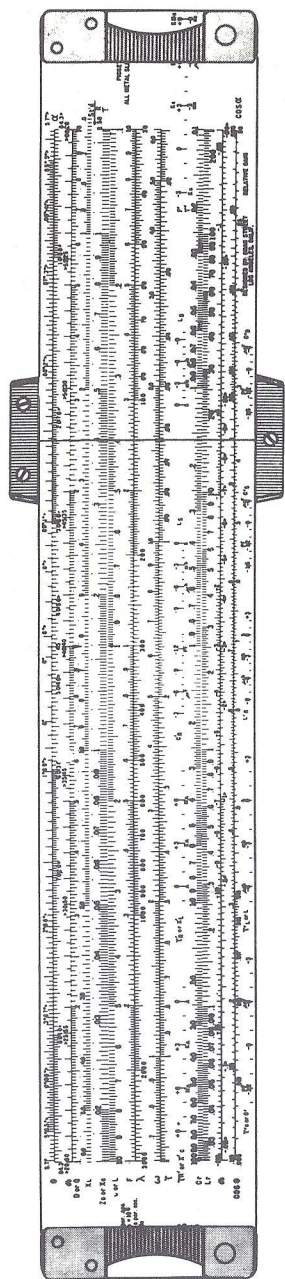


Figure 1b

Fig. 1b (back side) shows the electronic scales of the rule. The nomenclature of the parts of the rule referred to in this text are shown in the Figure.

In the center of the slide is a group of four scales. These are the principal scales of the rule. They represent time variable functions. These scales are interrelated and all calculations performed using this rule refer to them in one way or another. In the text, these scales will at times be referred to as F(T) scales, meaning "Function of Time." The first of the four scales is marked "F" and is a scale of frequency, i.e., the number of cyclic variations occurring in one second. This scale is calibrated in cycles per second. The second scale is that of the wavelength of an electro-magnetic signal in a vacuum. It is marked λ (Lambda). λ is related to the frequency F by the relationship:

$$\lambda = \frac{c}{F}$$

where c is the velocity of light in a vacuum. This scale is calibrated in meters $\times 10^6$. Such a large calibration is necessary because the values that would be directly related to F would be too large to place on the rule. The calibrated value of λ is that of a frequency read on the F scale in megacycles "Mc." The third of the F(T) scales is marked ω (Omega) and is the number of radians per second through which a vector would sweep if it made F complete rotations per second, i.e., $\omega = 2\pi F$. This is the basic F(T) of electrical relationships but frequency in cycles per second is more commonly used. τ (Tau) is the fourth of the F(T) scales and is related to ω by:

$$\tau = \frac{1}{\omega}$$

This is a scale for the unit of time and is calibrated in seconds.

With the aid of the electronic scales, three basic types of calculations and a number of variations of these three basics, can be performed. The CorL scale represents the capacitance in farads or the inductance in henrys respectively. These two scales, in conjunction with the X_c and the X_L scales, will solve for reactive impedance. The relationships for such solutions are:

$$X_c = \frac{1}{\omega C} \text{ and } X_L = \omega L.$$

The θ , $\cos \theta$ and db scales are used to solve the transfer functions of RL and RC circuits, both leading or lagging. These scales have two cycles in the length of the rule which puts the primary reference in the middle. This primary reference is marked with an arrow as shown in Fig. 1b. The F(T) scales should be read in reference to this arrow. Most attempts to design special rules for electronic or electrical work employ the use of many decade scales so that actual values can be read. However, the range of electronic systems is so great that it is not practical for such a rule to be read to any degree of accuracy. From the sub-audio work of the geophysicist, to the upper microwave and optical laser range, there is a ratio of about 10^{17} to 1. To resolve the difficulty, a set of decimal point finding scales were included. These scales are marked with primes (') and the symbols involved X'_L , X'_C refer to the type of problem being solved. At the lower interface of the slide and body of rule are the Cr and Lr scales. These, in conjunction with the F(T) scales, solve the resonant value of

LC combinations and the impedance and time delay distributed systems. These relations are:

$$\omega^2 = \frac{1}{LC} \text{ and } Z_c^2 = \frac{L}{C} \text{ and } T_0^2 = LC.$$

Lr and Cr are four cycle scales. Such a calibration is required to calculate the square root needed in the above relationships.

Just above the Xc scale there is a row of dots. These dots identify the value of the standard resistors in the 5 percent series. These values were assigned as standard by choosing the nearest two digit numbers that are whole number multiples of $10^{1/24}$, or the 24th root of ten. A similar row of dots above the X_L scale also represents the standard resistor series.

There is a further special reading that can be made in connection with the F(T) scales. This is the reading of the time period for any complete cyclic event for which the frequency (F) is known. The Cor L scale is calibrated as the reciprocal value of the F scale. The period T has the relation to F of $T = 1/F$.

The decimal point of the time period T is found on the τ' scale which is located on the right side of the slide. Any setting that finds the magnitude of F' on the F scale below on τ' scale will be the magnitude of T.

2.00 DECIMAL POINT FINDING SCALES

As mentioned in section 1.20, there is need in electronic work for a means of determining the order of magnitude of the answers found in circuit calculations. Order of magnitude usually means the power-of-ten as a multiplier. This term will have such a meaning throughout this text. The range of values encountered is so great that mental evaluation is strained to correctly determine the decimal location.

This situation is handled in the Model N-16 by a set of special scales on the slide and the body of the rule. These scales are marked with a prime (') such as C', X_c', L_r' --- etc. The following paragraph will briefly outline the principle of the decimal point finding scales. In the body of the text, the particular use of these scales will be covered for each type of

solution. The table below gives the relationship between the circuit values of a problem and the decimal point scales to be used.

CIRCUIT VALUES	SCALES
C, X _c , and F(T)	C', X _c ', and F'
C, R, and τ	τ_c' , τ_r' , and τ'
L, X _L , and F(T)	L, X _L ', and F'
L, R, and τ	τ_L' , τ_r' , and τ'
L, C, and F(T)	L _r ', C _r ', and F'
L, C, and τ	L _D ', C _D ', and τ'
L, C, and Z _s	L _s ', C _s ', and F'

The numerical values on the rule are calibrated in basic units such as henrys, farads, ohms, and cycles. For any numerical value read on the rule, there is some power of ten by which it can be multiplied that will express the desired value in basic units. As an example, assume that the value of a condenser being considered is 0.00235 μ f. Using the multiplier-of-ten system, this value of the condenser is written as follows:

$$0.00235 \mu\text{f} = 2.35 \times 10^{-9} = 0.00235 \times 10^{-6} \text{ farads.}$$

Thus the magnitude of the multiplier that will make the setting 2.35 for 0.00235 μ f equal farads is -9. Further, assume a 0.465 setting on the F scale to represent 46.5 Mc. To find the Magnitude of the multiplier that satisfies the setting, employ the following procedure:

$$\begin{aligned} 0.465 \times 10^2 \text{ Mc} &= 46.5 \text{ Mc} \\ 0.465 \times 10^2 \times 10^6 \text{ cycles} &= 46.5 \text{ Mc} \\ 0.465 \times 10^8 \text{ cycles} &= 46.5 \text{ Mc} \end{aligned}$$

+8 is the required multiplier. This process is used for all of the decimal finding scales. The units into which the scale readings are expressed are marked on the left end of the slide. The magnitudes to be applied to the F and ω scales are read on the F' scale at the right end of the slide. The reading is taken as the line value that is at the edge of the bridge (most commonly known as the end plate) as shown in Fig. 1b. The magnitudes for λ and τ are read on the τ' scale. In the case of λ , since it is calibrated in meters $\times 10^6$, this must be allowed for to find the true value in meters. To make this correction, simply multiply the value read on τ' by 10^6 .

Example 1: Assume that a reading of 326 on the λ scale and -7 on the τ' scale were taken. Then the correct magnitude would be $10^{-7} \times 10^6 = 10^{6-7} = 10^{-1}$, so λ in the final answer would be $326 \times 10^{-1} = 32.6$ meters.

Example 2: Assume that the arrow is set to 1.63 and this is to represent 1.63 KMc and an inductance of 0.415 μ h is meant at a setting of 4.15 on the L scale. We wish to know the magnitude to be applied to the reading of X_c .

$$1.63 \text{ KMc} = 1.63 \times 10^3 \text{ cycle and } 0.415 \mu\text{h} = 4.15 \times 10^{-7} \text{ henrys.}$$

To the inner side of the bridge (end plate) set +9 or KMc and set the hairline to -7 on the T_L' or L scale, read +2 on the X_L' scale directly above. This means that the value read for X_L' should be multiplied by 10^2 .

3.00 CAPACITIVE REACTANCE

The relationship to be solved in finding capacitive reactance is:

$$X_c = \frac{1}{\omega C} \quad \text{Eq. 1}$$

In analyzing a circuit, one is usually thinking in terms of frequency F. F can be substituted for ω in eq. 1 by use of the relationship: $\omega = 2\pi F$. This conversion is automatically taken care of by the calibration of the rule. Note, however, that the term ω , for angular velocity is the basic time function in network analysis. This will appear in many other circuit equations. The term "reactance" implies that the flow of current is impeded. But such impedance does not absorb any energy, since whatever is taken up by the condenser is returned to the system at a later time.

RULE: To find the capacitive reactance, set ω (angular velocity) or F (frequency) to the arrow in the middle of the D scale. Above the capacitance value on the CorL scale read the Capacitive reactance on the X_c scale.

EXAMPLE 1: What is the capacitive reactance X_c for a 0.7 farad condenser at a frequency of 3.5 cycles?

Move the slide so that 3.5 on the F scale is set to the arrow. Move the indicator so the hairline is at 0.7 on the CorL scale and on X_c read 0.065 ohms as shown in Fig. 3a. Thus, 0.065 is an ohmic value because the rule is calibrated in basic units of farads, henrys and ohms as mentioned earlier in section 1.20.

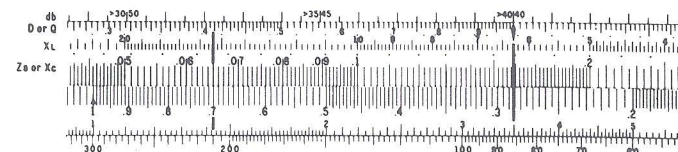


Figure 3a

EXAMPLE 2: To clarify the use of the decimal point finding system, let us assume that 0.7 stands for 0.7 μ f and 3.5 stands for 3.5 Kc. What will be the value of X_c ? Well, $0.7 \mu\text{f} = 0.7 \times 10^{-6}$ farads and $3.5 \text{ Kc} = 3.5 \times 10^3$ cycles. Move the slide until +3 (or Kc) is at the end plate, set the hairline to -6 (or μf) on the C' scale. Under the hairline on X_c' scale, read +3. The value of X_c will

then be $0.065 \times 10^3 = 65 \Omega$. So a $0.7 \mu f$ condenser will have an impedance of 65Ω at 3.5 Kc. We could have chosen any other set of units for the problem and found X_c . Let $C = 70$ Pf, (Pf stands for picofarads or micro-microfarads. This term is finding more acceptance than the older term of $\mu\mu f$. In this text, PF will be used as it is on the rule instead of $\mu\mu f$) and $F = 350$ Mc. The first step is to find the multiples-of-ten needed for the conversion.

$70 \text{ Pf} = 0.7 \times 10^{-10}$ farads and $350 \text{ Mc} = 3.5 \times 10^8$ cycles.

Set +8 on F' to the bridge, above -10 on C' read +2 on X_c' . Thus $X_c = 0.065 \times 10^2 = 6.5 \Omega$.

EXAMPLE 3: What value of condenser will have an impedance of $47 \text{ K}\Omega$ at 7 Kc?

Set 0.7 on F to the arrow. Move the hairline to 0.47 on X_c (the dot will help find this easily) and on CorL , read 0.484. For the decimal point, $0.7 \times 10^4 = 7 \text{ Kc}$ and $0.47 \times 10^5 = 47 \text{ K}\Omega$.

Set +4 on F' to the bridge and at +5 on X_c' read -9 on C' .

Therefore the value of the condenser is $0.484 \times 10^{-9} = 0.000484 \mu f = 484 \text{ Pf}$ (see Fig. 3b and 3c).

If F is set in Mc and CorL in Pf, then X_c is in ohms as read.

For practice, work out the following problems:

1. A 60 cycle, full wave rectifier has a ripple frequency at a fundamental of 120 cycles. What will be the impedance of a $40 \mu f$ condenser?

ANS. 33.2Ω .

2. In a system ω is given as 11,300, and a condenser is needed that will have an impedance of $1.2 \text{ K}\Omega$.

ANS. $0.074 \mu f$.

3. A transmitter is working on 21 meters. What is the value of a condenser that will have an impedance which matches the 50 ohm terminating resistor of a transmission line?

ANS. 224 Pf .

4. For a long wave receiver at an IF of 330 Kc what will be the impedance of a 156 PF condenser?

ANS. $3.1 \text{ K}\Omega$.

5. In a microwave system at 3.2 KMc what will be the value of X_c of a 1.5 Pf condenser?

ANS. 33.1Ω .

There are some easy rules to go by in finding the value of X_c under certain circumstances. Rules: If the value of the condenser set on CorL is in μf , then the value on X_c is in megohms for cycles on F . This means that whatever magnitude needs to be applied to F is the one to be applied to X_c with the sign changed, (-) for (+) and (+) for (-).

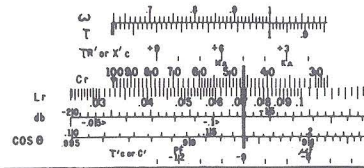


Figure 3c

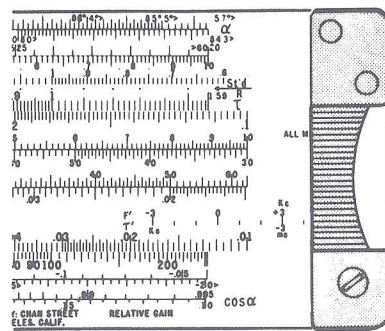


Figure 3b

4.00 INDUCTIVE REACTANCE

The solution for inductive reactance follows the same procedure as capacitive reactance except that the values on the C or L scales are in henrys instead of farads and the values of inductive reactance are read on the X_L scale in ohms (Red numerals).

The relationship to be solved here is:

$$X_L = \omega L \quad \text{Eq. 2}$$

Any function of time can be substituted for ω . This conversion will automatically be taken care of by the rule. Just set the desired value of F to the arrow and the equivalent F(T) will be set.

EXAMPLES:

(a) Find the inductive reactance of a 0.6 henry inductance at 5 cycles.

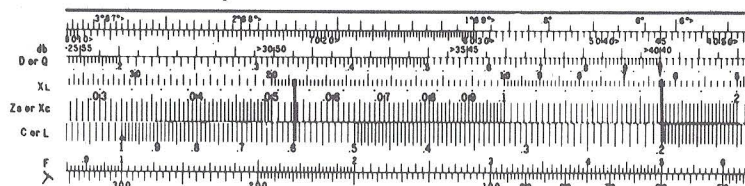


Figure 4a

Set 5 on the F scale to the arrow. Move the indicator to 0.6 on the L scale, and above read 18.8 on the X_L scale. (See Fig. 4a.) The values in the problem are those in which the rule is calibrated, i. e., ohms, henrys, and cycles. Therefore the answer is that read on the rule which is 18.8 Ω .

(b) Let us assume that the inductance in problem (a) was 6 millihenry and the frequency 50 Kc. What will be the value of X_L ? Well, 50 Kc = 5×10^4 cycles, so set +4 on the F' scale to the bridge and the indicator to -2 on the L scale ($6 \text{ mh} = 0.6 \times 10^{-2}$ henrys), then above on the X_L read +2. Hence, $X_L = 18.8 \times 10^2 = 1880 \Omega$. (See Fig. 4b.)

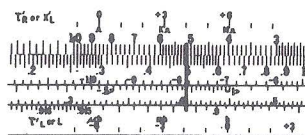


Figure 4b

From example (b) above, where only the decimal points of the units were shifted, it can be seen that a solution may be read on any part and the correct decimal point found. There is also a set of dots just above the X_L scale that give the value of standard resistors to aid in setting the indicator.

Below are a series of problems and answers that are included here for practice. These problems are chosen to cover the wide range of values that are encountered in electronic circuits.

1. Find X_C for a 0.01 μf condenser at 150 cycles.

ANS. 0.106 meg. ohms.

2. Find X_L of a 4h choke at 120 cycles.

ANS. 3010 Ω .

3. At what frequency will 65 Pf have a reactance of 0.47 megs? (Hint: 65 Pf = 0.65×10^{-10} f, Pf is marked on the C' scale and M Ω on the X_C scale.)

ANS. 5.21 Kc.

4. If the primary of an IF transformer has an inductance of 1.6 mh and a Q of 40, what will be its resonant impedance at 455 Kc?

ANS. 0.183 meg.

(HINT: Find X_L in the regular manner, 0.455 at arrow, indicator at 1.6 on C or L scale, read 4.57 on X_L , leave the indicator as set and move the slide till the 1 on the L scale is at the hairline, then move the indicator to 4 on the L scale and read 18.3 on the X_L scale. This last operation multiplied X_L by 40 to give Z the impedance. $X_L Q = Z$, the resonant impedance.)

5. The output impedance of an oscillator is 500 Ω and a condenser is to be put in series with it so that at the lowest frequency of the instrument which is 20 cycles, its reactance is at least 1/10 the output impedance. What is the smallest value that can be used?

ANS. 160 μf .

5.00 LC RESONANCE

At the bottom of the rule, there are two scales marked Cr and Lr. These scales are used to solve the combination of inductance and capacitance that will be resonant at some frequency. This resonant frequency may be read as frequency F, angular velocity ω or as wave length λ . Notice that the Cr and Lr scales vary through four orders of magnitude across the length of the rule instead of two cycles as for the F(T) scales. This establishes the square root relationship required for the calculation. The equation to be solved for either series or parallel resonance is:

$$\Omega^2 = \frac{1}{LC} \quad \text{Eq. 3}$$

Set the slide in any position and examine the relationship between the Cr and Lr scales. All values on the Cr scale represent capacitance, values that will resonate with the opposite value of inductance.

EXAMPLE: Set 0.2 on Cr to 3 on Lr. Note 0.01-60, 0.04-15, 0.5-1.2, and 8-0.075 etc. With the indicator find the value of F that is at the arrow, read 2.06. This means that all the values of C and L read opposite each other on Cr, and Lr will resonate at a frequency of 2.06 times some power of 10, or 2.06×10^x where X will be determined by the values assigned to C and L. Let us assume that the 0.1 on Cr is 0.1 μf and the 6 on Lr is 60 mh. To find the decimal point note that $0.1 \times 10^{-6} = 0.1 \mu\text{f}$ and $6 \times 10^{-2} = 60 \text{ mh}$. Set the indicator to -2 on the L'_r scale at the very bottom of the rule and move the slide till -6 or μf on Cr is at the hairline. At the bridge will be +3 or Kc, meaning that the frequency is 2.06 Kc.

Because of the finding of the square root, the values chosen for the powers of ten for Cr and Lr must add to an even number. This can be shown if the values of 0.1-6 are meant a 1 μf condenser and a 60 mh inductor. $1 \mu\text{f} = 0.1 \times 10^{-6}$ farads and $60 \text{ mh} = 6 \times 10^{-2}$ henrys, the sum is -7 odd. Setting the indicator to -2 on L'_r and -5 on C'_r the bridge comes halfway between +3 and +4. This shows that 2.06 is not the correct reading for this combination.

RULE: Wherever the sum of the powers of ten add to an odd number, move the slide one scale cycle in either direction along the Cr or Lr scales. In the above problem, let us move the slide to the right so that the 1 is over the 6 on the Lr scale

and at the arrow read 0.65. We have a decimal point power of -2 for L and -6 for C. Set -2 on L'_r and -6 on C'_r and read +3 on F. Therefore the resonant frequency is 650 cycles.

One further example will be given. Assume that a 4.3 mh coil is desired to be resonant at 23 Kc. For the coil $4.3 \text{ mh} = 4.3 \times 10^{-3} \text{ h}$ and $23 \text{ Kc} = 2.3 \times 10^4 \text{ cycles}$. Set +4 on F' to the bridge and the indicator to -3 or mh on L'_r and read -7 on C'_r . Put 2.3 on F to the arrow and above 4.3 on Lr read 0.112. Therefore $0.112 \times 10^{-7} = 0.0112 \mu\text{f}$.

1. An oscillator coil has an inductance of 3.2 mh and a Q of 35. What value condenser will make it resonate at 150 Kc and what is the tank circuit impedance?

ANS. $C = 352 \text{ Pf}$ and $Z = 105.5 \text{ K}\Omega$.

2. A transmitter's output tank has an inductance of 1.6 μh and it is to resonate when the tuning condenser is set at 50 Pf and the output has a wave length of 21 meters. What is the size of the padding condenser required?

ANS. 28 Pf.

3. The coil of a single tuned filter was found to work when tuned with a 0.047 μf condenser. The coil has an inductance of 0.0245 h. What is the resonant frequency of the circuit?

ANS. 4.7 Kc.

6.00 TRIGONOMETRIC FUNCTIONS

As mentioned in Sec. 1.10, on the math side of the rule there are scales for hyperbolic functions. These are located on the upper part of the rule while the circular functions appear on the slide. The latter scales are used in solving geometric configurations. Section 6.10 covers the use of the circular scales as they appear on the N-16. The hyperbolic functions are covered in Section 6.30.

Solution of Right Angle Triangle: The trigonometric scales of the N-16 have been reversed so that they run from right to left instead of the conventional left to right direction. This change was made to simplify the solution of right angle triangles. In the conventional layout the trig functions read directly against the C scale. About 25 years ago, slide rules began to be produced with a DI scale so that triangles could be solved in one setting. This has been, more or less, the standard since that time. When the trig scales are inverted, a single setting solution of triangles can be made on the D and C scales. This latter method is faster and does not require as much use of the indicator.

As an example of how this is accomplished, let us use the standard 3-4-5 triangle (this is always a good method of figuring out how to handle an unknown rule), $5^2 = 3^2 + 4^2$. Suppose we know the length of the two legs of the right triangle and wish to find the length of the hypotenuse. Set the left index of the C scale to 3 on D. Move the indicator to 4 on D and read 36.90° on T. Without moving the slide, shift the indicator to 36.9° on S and at the hairline on D read 5. (See Fig. 6a.) This has solved for two unknowns, the angle opposite the shortest leg of the triangle and the hypotenuse. This process should be practiced thoroughly since it is used a great deal in network analysis.

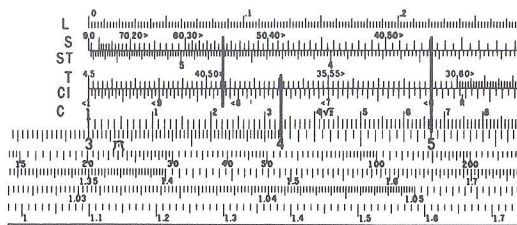


Figure 6a

PRACTICE PROBLEMS

	Base	2nd Leg	Angle°	Hyp.
1.	5.00	2.00	21.8	5.39
2.	3.50	1.50	23.2	3.31
3.	43.0	32.6	37.2	54.0
4.	21.0	6.50	17.2	22.0*
5.	110.0	92.0	39.9	143.5

*Hint, set right index to 6.5.

6.11 Rules of Procedure

Some very simple rules can keep the procedure in mind.

1. Always visualize the triangle in the first or fourth quadrant, i.e., $+X$ and $\pm Y$. One is always solving for the angle the hypotenuse makes with the X axis.
2. Always set the smaller (in magnitude) of the two values to the left or right index of the C scale.
3. If the smaller of the two is along the Y axis, read the angle on the black numbers of the T scale.
4. If the smaller value is on the X axis, read the angle on the red numerals. These will be greater than 45° .
5. When solving for the hypotenuse, always use the same colored numerals on the S scale as were used on the T scale.

See Fig. 6b for the diagram on which the above rules are based.

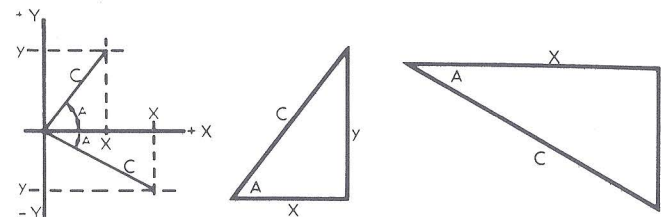


Figure 6b

Keeping the above rules in mind, work out the following:

X	Y	A°	C
23.6	+8.65	+20.0	25.3
54.5	+134.0	+67.9	145.0
71.5	-12.7	-10.09	72.6
266.0	+17.6	+3.80	266.0
47.5	-169.0	-74.3	176.5
12.4	+332.0	+87.86	332.0*

*Hint: $90 - 2.14 = 87.86$.

If the numerical values of the trig functions are needed they can be read by setting the hairline to the function desired and then reading its value on the CI scale. See Fig. 6a, $\sin 55^\circ = 0.819$.

6.20 Cosine of Small Angles

There are times when the cosine values of small angles are required. Examining the left end of the sine scale, it can be seen that the graduations become crowded together as they approach the left index. The red numerals identify the cosine value while the black the sine values. The trigonometric relation:

$$\cos \theta = \frac{\sin \theta}{\tan \theta} \quad \text{Eq. 4}$$

shows that the difference of two distances on the D or C scales corresponds to the logarithm of the quotient of the two numbers represented, i. e., the difference between the same angle graduations on S and T scales equal the cosine of the same angle. This relationship allows us to set, with accuracy, the cosine of small angles. Examples:

1. Assume that it is desired to find $X = Y \cos 14.4^\circ$, for $Y = 3.56$. Set 14.4° on S to 3.56 on C. Without moving the slide read 3.445 on C at 14.4° on T. This has solved for the desired value. Notice that this is the same method used in solving for the hypotenuse of a right angle triangle. The setting for this example is shown in Fig. 6c.
2. If we had wished to divide by the cosine, the tangent would be set to the dividend and the answer read at the sine. Using the same values as at example (a), to 3.56 on D set 14.4° on T and at 14.4° at S read 3.68 on D. This has performed the desired division.

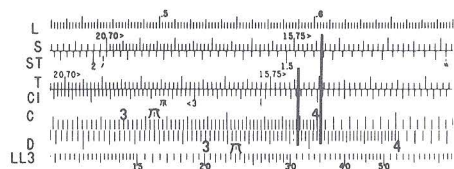


Figure 6c

6.21 Sine of Large Angles

The same technique used to handle the cos of small angles can be used for the sine of large angles. The difference between the red numerals on the S and T scales is represented by:

$$\sin \theta = \tan \theta \cos \theta \quad \text{Eq. 5}$$

Rule: To multiply a value by the sine of a large angle add the length between the red angular calibrations on the S and T scales to the value and read the result. To divide, subtract the difference in length.

In Fig. 6c, the setting shows 3.56 being multiplied by the cos of 14.4° and by the sine of 75.6° .

6.30 Hyperbolic Functions

On the upper edge of the body of the rule on the math side, there are three scales marked SH, SH2 and TH. These are the hyperbolic scales. These functions are based on a hyperbola. In part 6 of the Manual on the Dual Base Log Log rule that accompanies the N-16 is an extensive discussion of these functions. Their use in electrical engineering is most common in connection with distributed systems such as transmission lines. To delineate the subject of distributed systems is beyond the scope of this manual. For those who wish to study this phase of network analysis more thoroughly, two references are recommended. Section 16 in Electronic Designers' Handbook by Laudel, and Section 3, page 172 of Radio Engineers' Handbook by Terman.

The discussion in the Dual Base Log Log manual is based on the layout of Model N-4 slide rule where the hyperbolic scales are on the slide. In the N-16, there was no room for this configuration since all the conventional scales had to be placed on one side of the rule. To convert the explanations given for the N-4 and make them applicable to the N-16, it must be borne in mind that the values of the hyperbolic functions appear on the D scale instead of the C scale. To show how this conversion is made, refer to page 68 of the Dual Base Log Log manual, examples a, b, c, and d.

- a) Solve for Y in $Y = 24.6 \sinh 0.35$. Set hairline to 0.35 on SH₂ and set left index of C to hairline, at 24.6 on C read 8.79 on D.
- b) Find Y in $Y = 86.4 \tanh 0.416$. Set right index of C to 0.416 on TH using indicator, at 86.4 on C read 34.0 on D.

- c) Find $Y = 77.3 \cosh 1.26$. The relation between $\cos h$, $\sin h$, and $\tan h$ have the same form as $\sin$, $\cos$, and $\tan$ (see Sec. 6.21). Using the indicator set 77.3 on C to 1.26 on TH and at 1.26 on SH, read 147 on C.
- d) Solve for Y, where $Y = 17.9 \sin h 0.317 \times \sin 22^\circ$. To 0.317 on SH_2 set 22° on S, at 17.9 on C read 2.161 on D.

Some explanation is in order on example (d) above. Just as multiplication by a reciprocal is equivalent to division, so is division by a reciprocal equivalent to multiplication. Since the trig scales on the N-16 are inverted, their values on the C and D scales are the reciprocal of their actual values. By placing $\sin 22^\circ$ to $\sinh 0.317$ the two functions were multiplied and the result appeared at the left index of C as 0.1207. Since we now wish to multiply this value by 17.9 (as required by problem), we simply read the answer on D at 17.9 of C without moving the slide or the indicator. It is not even necessary to read the value 0.1207; the intervening step is just part of the solution.

If one remembers that the values of the hyperbolic functions appear on the D scale, and the reciprocals of the trig functions on the C scale, there should be no difficulty in applying the very clear explanations given by Dr. Hartung to the N-16.

7.00 SPECIAL SOLUTIONS FOR CIRCUIT ANALYSIS

There are a number of special factors that are used in circuit analysis. The most common of these and the method of using the N-16 for their calculation is given below.

7.10 Time Constants

In a series circuit which consists of a resistor and a condenser as shown in Fig. 7a, a suddenly applied voltage will cause a current to flow that changes with time.

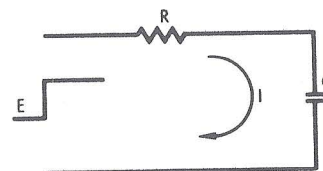


Figure 7a

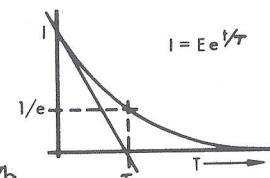


Figure 7b

The relation of this current to time is given by the exponential function,

$$I = \frac{E}{R} e^{-\frac{t}{\tau}} \quad \text{Eq. 6}$$

Where τ represents the value of RC , this is called the "time constant" and when t reaches a value equal to τ , the expression becomes $I = \frac{E}{R} \frac{1}{e}$, where e is the base of the "Natural logarithms." Just as a matter of interest, the approximate value of e is 2.71828 and $\frac{1}{e} = 0.36787$. For this type of circuit, the value of RC is measured in seconds and $RC = \tau$. This is how long it takes the current to decay to a value equal to $\frac{1}{e}$ or 0.367 of its original value. The value of I at $t = 0$ is $I = \frac{E}{R}$, where E is the height of the step of voltage applied and R is the resistance in the circuit. The time constant of this type of circuit is an indication of how rapidly it can respond to a change.

RULE: To find the time constant of an RC combination, set the value of the condenser on the CorL scale to the value of R on the X_C scale and at the arrow read the value of τ_c on the τ scale. Note that this same value can be read at the small arrow in the middle of the CorL scale on the X_C scale.

EXAMPLE: Find the time constant of a $0.026 \mu\text{f}$ condenser and a $12 \text{ K}\Omega$ resistor. Set 2.6 on CorL to 0.12 on X_c and at the arrow read 0.312. To find the magnitude that applies, we have $2.6 \times 10^{-8} \text{ f} = 0.026 \mu\text{f}$ and $0.12 \times 10^5 \Omega = 12 \text{ K}\Omega$. Set +5 on τ_R' to -8 on τ_C' and at the bridge read +3 or ms on τ' . Then the answer is 0.312 ms. This means that a suddenly applied step of voltage to such a circuit would cause the current to drop to 0.367 of its original value in 0.312 ms.

PRACTICE PROBLEMS AND ANSWERS:

C	R	τ_c
20 μf	150 $\text{K}\Omega$	3.0 sec.
500 Pf	47 $\text{K}\Omega$	23.6 μs
0.047 μf	51 $\text{K}\Omega$	2.4 ms
6.5 Pf	470 $\text{K}\Omega$	30.6 ns

In solving all the above time constants, bear in mind that when the rule is set to the combinations given, any other pair of values of R and C that are opposed on the CorL and X_c scales will give the same answer as long as their values are properly assigned.

For RL combinations, the time constant is given by:

$$\tau_L = \frac{L}{R} \quad \text{Eq. 7}$$

The method of solving for τ_L is essentially the same as τ_C .

RULE: The designated combination of R and L are set opposite each other on the X_L and CorL scales and the answer is read on the τ scale or at the small arrow on the X_c scale.

For practice, solve for the underlined factors in the following problems:

<u>L</u>	<u>R</u>	<u>τ_c</u>
4.5 h	500 Ω	9.0 ms
160 mh	6200 Ω	25.8 μs
14 μh	40 Ω	350.0 ns
4 mh	75 Ω	53.4 μs

7.20 Finding db on the Log Log Scales

The term "Bel" was originally used to define the power ratio between the input and output of a circuit where both had a 500Ω

impedance. It was defined as $\text{Bel} = \log_{10} \frac{P_2}{P_1}$. This was found to be too large a value so a smaller one was used, the "decibel," which is defined as $\text{db} = 10 \log_{10} \frac{P_2}{P_1}$. The success of this definition was so great that it came to identify, not only power ratios, but voltage ratios as well. Where a voltage ratio is used, it has the relation $\text{db} = 20 \log_{10} \frac{V_2}{V_1}$. As will be discussed in later sections, in certain operations of the rule, the db ratio is read directly on the electronic side. There is however, a method of using the Log Log scales that will solve for db over a very large range.

RULE: To find the power ratio in db, set the slide so that the left or right index of the C scale is at 10 of LL3. Move the hairline to the ratio on the appropriate LL scale and read the value of db under the hairline on C scale.

EXAMPLE: Find the db value of the ratio 50. Set the left index of C scale over 10 of LL3. Move the hairline to 50 on LL3 and read 17 on C directly under the hairline. This means that if the ratio between the input and output power is 50:1 it will have a db value of 17. This setting is shown in Figure 7c.

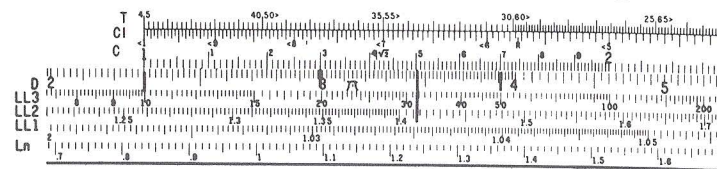


Figure 7c

Several readings in addition to the set point are shown in the figure; 13 db = 20:1, 1.5 db = 1.413:1, -.15 db = 0.0352:1 and 17 db = 50:1.

PRACTICE PROBLEM:

Find the db value for the following ratios:

RATIO	db
15:1	11.78
20:1	13.02
40:1	16.03
62:1	17.97
150:1	21.8

If a voltage or current ratio is desired in db, set 2 on the C scale to 10 on LL3 scale and read as above for power. The voltage values will be twice those for power.

8.00 COMPLEX IMPEDANCE

The impedance of AC circuits cannot be solved in the straight-forward methods used for DC. Since the current through both an inductance and a condenser flows at a time $1/4$ of a cycle different than the voltage across it, another method of solution is required. This is called complex algebra. See part 3 of the Dual Base Log Log manual. The N-16 is designed to solve this type of problem directly.

8.10 RC Series

Fig. 8a is a graphical plot of the method used. Fig. 8a is the plot of the circuit in Fig. 8b. The distance along the horizontal line is the value of R and the downward direction direction is the reactive impedance of C. Such a plot could be made by finding X_c on the rule and solving for Z and θ by trigonometry. With the N-16 this is not necessary; an equation of the type shown in Fig. 8c can be solved directly. To illustrate how

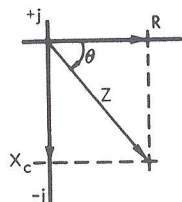


Figure 8a

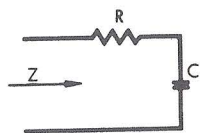


Figure 8b

$$Z = R - jX_c = |Z| \angle -\theta$$

Figure 8c

this is done, let $R = 12 \text{ K}\Omega$ and $C = 0.022 \mu\text{f}$. What is the phase shift θ and the magnitude of Z at 400 and 1000 cycles? $12 \text{ K}\Omega = 0.12 \times 10^5 \Omega = 12,000 \Omega$ and $0.022 \mu\text{f} = 0.22 \times 10^{-6} \text{ f} = 2.2 \times 10^{-8} \text{ f}$. Set -8 on C' to +5 on X_c' and read +3 on F' . Set 2.2 on C scale to 0.12 on X_c and the hairline to 0.4 on F. On the bottom half of the θ scale read 56.5° (black numerals). This will be the phase angle as shown in Fig. 8a and will be negative for capacitive reactance (Z has rotated clockwise). Without moving the hairline make note of the value 0.552 on the $\cos \theta$ scale. Now move the indicator to 1 on F for the 1Kc reading. θ is 31.2° and $\cos \theta$ is 0.855. See Fig. 8d for those two settings.

In network problems, it is often necessary to solve for the value of Z. This value could be found by using either the math side or the electronic side of the N-16. From Fig. 8a, it is seen that $Z = R / \cos \theta$. Set the arrow just above the 1 in the

middle of the CorL scale to R (0.12) on X_c and above 0.552 on CorL read 0.217 on X_c . The decimal point here can be set by inspection. Since we are dividing by approximately 0.5, the answer will be twice as big or $21.7 \text{ K}\Omega$.

There is another factor often used in network analysis, known as The Dissipation Factor. It is symbolized by "D" and $D = \omega RC$. Since $X_c = \frac{1}{\omega C}$, $D = \frac{R}{X_c}$ and from Fig. 8a, $D = \cos \theta = \tan \alpha$. This term is a quality factor and indicates the degree with which a circuit approaches a pure reactance. The smaller "D" the less energy is dissipated as IR losses in the system. The value of D can be read for any combination of C and R for any frequency in question. In Fig. 8d notice that for 400 cycles $D = 0.661$ and at 1 Kc, $D = 1.66$. A "D" of unity means a phase shift of 45° and $X_c = R$. The term "D" is not used as often as the quality factor "Q" that applies to inductive systems. This unit will be covered under RL circuits.

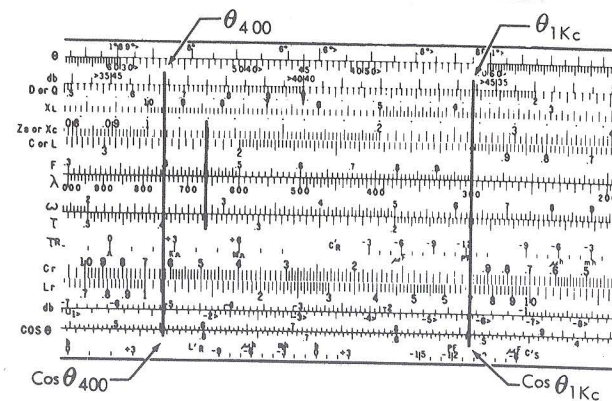


Figure 8d

The N-16 is designed to read phase angles from 0.6° and 89.4° . The $\cos$ of θ can be read only from 5.7° to 84.3° . At large angles, Z will equal X_c within 0.6% accuracy while at small angles, Z will equal R within the same accuracy. The relation $Z = R - jX_c$ is called the complex form whereas $Z = |Z| \angle -\theta$ is the polar form. It is often necessary to convert back and forth between these two forms. This conversion could be done either on the electronic side as shown above, or the math side as discussed in section 6.00.

PRACTICE PROBLEMS:

1. What is the series impedance of a $47 \text{ K}\Omega$ resistor and a $0.0027 \mu\text{f}$ condenser at 6 Kc?

ANS. $Z = 48 \text{ K}\Omega \angle -11.8^\circ$.

2. What is the value of Z when $C = 1 \mu\text{f}$ and $R = 200 \Omega$ with $F = 120$ cycles?

ANS. $Z = 134 \Omega \angle -81.45^\circ$.

3. A tube is loaded with $27 \text{ K}\Omega$ resistor in series with a $0.0114 \mu\text{f}$ condenser. What is the load impedance at a frequency of 400 cycles?

ANS. $Z = 44 \text{ K}\Omega \angle -52.4^\circ$. (See hint below.)

4. The output of a high frequency generator is loaded with a series combination of a 30Ω resistor and a 7 Pf condenser. What is the load impedance at 1.56 KMc ?

ANS. $Z = 33.4 \Omega \angle -25.9^\circ$.

HINT: It is important to remember that at any setting of slide, all the resistance values on the X_c scale opposite capacitance values on the C scale will have a phase shift as shown for any frequency to which the hairline is set. Since this is a series circuit, its impedance and phase shift will decrease with increasing frequency. Therefore the θ scale (which decreases going to the right as the frequency increases) should be read.

8.20 RL Series

The impedance of a series RL (resistance-inductance) is handled on the rule in much the same manner as the RC circuit was. In this case, however, the values on the CorL scales are values of inductance and the resistance component is read on the X_L scale. Fig. 8e is the complex diagram for a series RL combination.

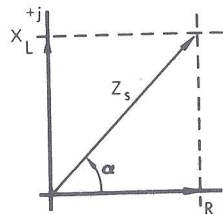


Figure 8e

Fig. 8e shows the value of X_L measured upwards and is prefixed with $+j$. The phase angle is marked α to coincide with the marking on the rule. The values of X_L and its associated phase angle α are marked in red. Notice that α increases with frequency, i.e., increases going to the right.

Just as in the RC system, there is a quality factor that applies to inductances. A coil is usually drawn schematically

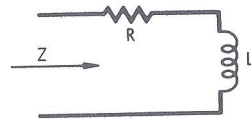


Figure 8f

$$Z = R + jX_L = |Z| \angle +\alpha$$

Figure 8g

with its losses as a symbolic resistance in series with the inductance. Often there is a real resistance added and this is lumped with the inherent losses of the coil. Such a circuit is diagrammed as in Fig. 8f where $Q = \tan \alpha = X_L/R$. For any pair of values of L and R on the L and X_L scales, Q may be read for the desired frequency. There is no provision on the rule to determine the decimal point for Q since it can readily be done by inspection. Q is unity at the frequency where $X_L = R_L$, the loss resistance of the circuit. The relationship $Q = \omega \frac{L}{R}$ shows that Q varies directly as ω . From this it can be

seen that one can read Q to any value by applying a multiple of ten to the frequency and the same multiple to Q .

EXAMPLES:

a. A coil of 5 mh has a loss of 20Ω . What is the Q for this coil at 15 Kc ?

Set the rule as indicated in Fig. 8h. For this combination the frequency multiplier is 10^3 , so the line on the F scale is at 150 cycles, and the Q reading is 0.235. Therefore 100×150 cycles = 15 Kc . Q is multiplied by the same factor, i.e., $100 \times 0.235 = 23.5$. If the losses were still 20 ohms at 150 Kc and the inductance the same, then Q would be 235.

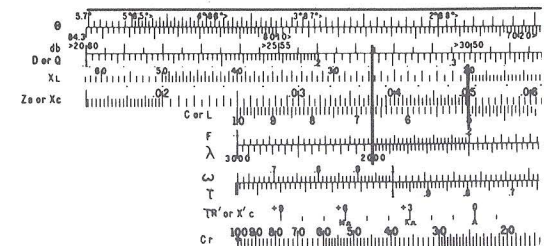


Figure 8h

b. Assume that R in Fig. 8f is 18Ω and $L = 0.75 \text{ h}$. What will be the complex impedance at 6 cycles? To 18 on the X_L scale set 0.75 on the CorL scale, move the hairline to 6 on the F scale, and on the α scale read 57.5° . From Fig. 8e it can be seen that $Z_s = R/\cos \alpha$. The division of R can be performed on several scales of the rule, but the most direct method is to set 0.538 ($\cos \alpha$) on CorL to 18 (R) on X_L and at 1 on CorL read 33.4 on X_L . Since the values used in this example are the ones in which the rule is calibrated the result is $Z_s = 33.4 \Omega \angle +57.5^\circ$.

NOTE: The cosine scale is calibrated in both directions. The scales marked in black refer to the angles numbered in black (θ). The red cos scales refer to the angles marked in red (α).

PRACTICE PROBLEMS:

1. In Fig. 8f, $R = 520 \Omega$ and $L = 0.24 \text{ mh}$. What is the impedance (Z_s) of this combination at 150 Kc?

ANS. $Z_s = 567 \angle +23.5^\circ$.

2. The collector of a transistor is loaded with 42 mh coil in series with a 680Ω resistor. What is the load impedance at 6 Kc?

ANS. $Z = 1720 \Omega \angle +66.7^\circ$.

3. A network requires an RL series combination that will give a 16° phase shift at 2 Kc and have an impedance of not more than 500Ω . What values of R and L will meet this requirement as close to the 500Ω limit as practical?

ANS: $R = 470 \Omega$, and $L = 10.65 \text{ mh}$.

HINT: Set hairline to 16° on α scale, read $\cos \alpha$ and multiply it by 500. The answer is a little more than 470Ω . Set 0.2 on F to 16° on α read any combination of R and L at this setting; all of these will give the same phase shift.

8.30 RC and RL Parallel

A third and final type of circuit in connection with RC and RL combinations is the parallel combination of a resistor with a condenser or an inductor. The method used on the rule will be explained in connection with Fig. 8i. The impedance Z_s is drawn in the same manner as was done in Fig. 8a. A line is also drawn from the value of R on the abscissa to that of X_c on the ordinate. The triangle thus formed is the same as the one when Z_s is the hypotenuse and therefore θ is simply in the opposite corner. α on the rule is $90 - \theta$ so the angle this line makes with the $-j$ axis is α . An additional line is drawn from the origin to the diagonal line and is normal to it. The length of this line is the magnitude of the parallel impedance, Z_p , and by similar triangles α is the phase angle. From the relationship $\cos \alpha = Z_p / R$, it is apparent that if α is found then Z_p can be determined by $Z_p = R \cos \alpha$.

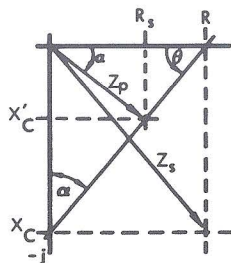


Figure 8i

EXAMPLE:

A tube is loaded with 27 K resistance in parallel with a $0.011 \mu\text{f}$ condenser. What is the load impedance at a frequency of 400 cycles? Here $\alpha = 90 - \theta = 90 - 52.4^\circ = 37.6^\circ$. $\cos 37.6^\circ = 0.793$.

$$Z_p = R \cos \alpha = 27 \text{ K} \times 0.793 = 21.5 \text{ K} \Omega \angle -37.6^\circ.$$

The method is identical in an RL combination except the diagram would be drawn on the $+j$ axis. For the RL parallel circuit the roles of θ and α are reversed, i.e., one reads the theta scale for the phase angle and then finds Z by $Z = R \cos \theta$.

PRACTICE PROBLEMS:

1. A $0.5 \mu\text{f}$ condenser is paralleled by 510Ω resistor. What is the impedance of this combination at 400 cycles?

ANS: $Z_p = 430 \Omega \angle -32.6^\circ$.

2. The input capacitance of a vacuum tube is 65 Pf and the grid resistance is 479 K. What will be the loading on the driving source at 20 Kc?

ANS: $Z = 119 \Omega \angle -75.4^\circ$.

3. A 0.0248 mh coil shunted by a 10 K resistor is in the plate circuit of a pentode. What is the plate loading at 45 Kc?

ANS: $Z = 5.73 \text{ K} \Omega \angle +55.0^\circ$.

4. The load on a power supply is a 3.5 h inductance paralleled by a 2500Ω resistance. What impedance will this offer to the ripple frequency of 120 cycles?

ANS: $Z = 1785 \Omega \angle +43.4^\circ$.

If it is desired to find the series equivalent of a parallel circuit, Fig. 8i shows how this is done. The two dashed lines at the end of Z_p indicate these values where they intersect the $-j$ and R axis. The values are marked R_s and X'_C . $R_s = Z_p \cos \alpha$ and $X'_C = Z_p \sin \alpha$. There are a number of trigonometric relationships linking the parallel values with their equivalent series values and any of these can be used.

9.00 NETWORK SOLUTIONS

The technique of network analysis is a major subject in electronics and can only be dealt with here in a very elementary form.

For those desiring to go more deeply into this subject an excellent text is Network Analysis by M. E. Van Valkenburg published by Prentice-Hall.

The N-16 has many advantages in the stepwise calculations of network problems. It would be almost impossible to present all the uses the rule can be put to in this type of analysis. However, a number of examples will be covered and familiarity with the rule will bring to light further applications.

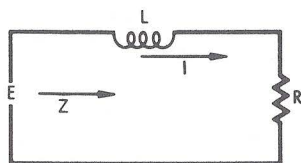


Figure 9a

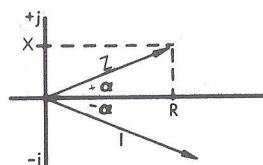


Figure 9b

EXAMPLES:

a. Solve for the current and voltage drops in the circuit shown in Fig. 9a.

Let the applied voltage E be 73.0 volts at 650 cycles, $L = 0.04$ h and $R = 400 \Omega$. Solve for X_L and Z . Here Z is found just as was done in the series RL circuit of section 8.20. Therefore

$$X_L = 163 \Omega \angle +90^\circ, \quad Z = 432 \Omega \angle +22.2^\circ.$$

Ohms Law is really a vector relation where $I = \frac{E}{Z}$, hence for this circuit

$$I = \frac{73 \text{ v} \angle 0^\circ}{432 \Omega \angle +22.2^\circ} = 0.169 \text{ amp} \angle -22.2^\circ$$

RULE: To divide two vectors in complex algebra, put them in polar form, divide their magnitudes and subtract the phase angle of the divisor from that of the dividend.

$$\frac{73}{432} = 0.169 \text{ and } 0 - (+22.2) = -22.2^\circ.$$

Again from Ohms Law the voltage across any element is $E = IZ$.

RULE: When multiplying, have the vectors in polar form, multiplying their magnitudes and add their phase angles.

$$E_L = IX_L$$

$$163 \times 0.169 = 27.5$$

$$+90 + (-22.2^\circ) = +67.8^\circ$$

$$E_L = 27.5 \text{ v} \angle +67.8^\circ \quad E_R = IR$$

$$400 \times 0.169 = 67.5 \text{ and } 0 + (-22.2^\circ) = -22.2^\circ$$

$$E_R = 67.5 \text{ v} \angle -22.2^\circ$$

b. The circuit to be analyzed is shown in Fig. 9c.

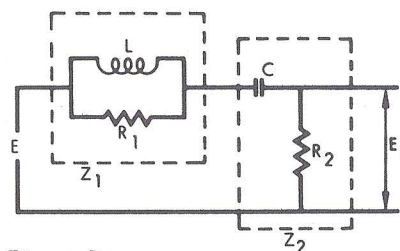


Figure 9c

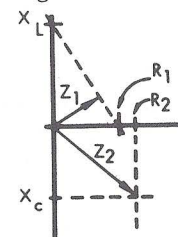


Figure 9d

The parallel RL and the series RC combinations of Fig. 9c have been grouped as lumped impedances Z_1 and Z_2 respectively. These are solved for as previously discussed in sections 8.30 and 8.10. Let $E = 112$ volts at 2.3 Kc, $L = 177$ mh, $R_1 = 1600 \Omega$, $C = 0.37 \mu\text{f}$, and $R_2 = 2200 \Omega$. Find the voltage developed across R_2 . Using the method of finding the parallel impedance of an RL combination $Z_1 = 1355 \Omega \angle +32.0^\circ$. Using the method for RC series combination $Z_2 = 2890 \Omega \angle -40.5^\circ$. These are shown in Fig. 9d. To add those two impedances ($Z_1 + Z_2$), they should be put in the complex form $\alpha + jb$. In either case it only requires the solution of a right angle triangle by the technique in section 6.10. Solving for Z_1 , the hypotenuse 1355 is placed at $\sin 32.0^\circ$. Set the hairline to 1355 on D and move the slide until 32.0° on S is at the hairline. Move the hairline to 32.0° on T and read 1150 on D and at the right index of C, on D read 720. Therefore, $Z_1 = 1150 + j720$. To solve for Z_2 , set 2890 on D to 40.5° on S. Move the hairline to 40.5° on T and read 2200 on D. At the left index of C read 1875 on D. Therefore, $Z_2 = 2200 - j1875$. The equivalent circuit of Z_1 and Z_2 is that of two impedances in series. To find the sum of these two add the real and the imaginary parts of each respectively, i.e., $Z_0 = (1150 + 2200) + j(720 - 1875) = 3350 - j1160$. Since we now want to find the current, this will

require division, the new impedance Z_0 must be converted back into a polar form to make the division possible. To do this set the left index of the C scale to 1160 (smaller of two values) on D. Move the hairline to 3350 on D and read 19.1° on T (black numerals). Shift the hairline to 19.1° on S and read 3550 on D. This gives Z_0 , in polar form, $Z_0 = 3550 \Omega \angle -19.1^\circ$. (A negative phase angle is a result of $-j$ in the complex form.)

$$\text{Now } I = \frac{E}{Z} \text{ or } I = \frac{112}{3550 \angle -19.1^\circ} = 0.0316 \angle +19.1^\circ$$

Note: $0 - (-19.1) = +19.1^\circ$

Then the current

$$I = 31.6 \text{ ma} \angle 19.1^\circ \text{ and } E_R = 2200 \times 0.0316 \angle 19.1^\circ = 69.5 \text{ v} \angle 19.1^\circ$$

This may seem like a long process to one not already familiar with this type of calculation. But with a little practice it goes quite rapidly, approximately $3\frac{1}{2}$ to 4 minutes for the above problem.

c. The stepwise answers will be given for this example. The rule technique will be left out for the reader to practice.

In Fig. 9e, solve for the complex voltages that appear across the portions of the circuit as shown.

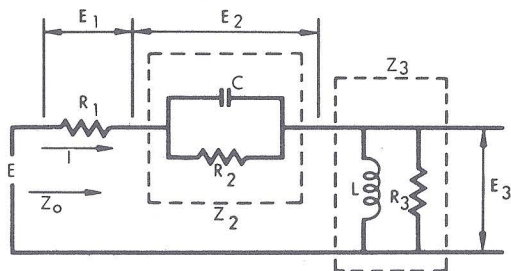


Figure 9e

The circuit values are: $E = 140 \text{ v}$ at 2.45 Kc , $R_1 = 470 \Omega$, $R_2 = 2700 \Omega$, $R_3 = 2000 \Omega$, $C = 0.025 \mu\text{f}$, and $L = 118 \text{ mh}$.

ANSWERS:

$$Z_2 = 1250 \angle -62.4^\circ, Z_3 = 1715 \angle +31.0^\circ, Z_0 = 2151 \angle -5.19^\circ,$$

$$E_1 = 26.2 \angle +5.19^\circ, E_2 = 69.6 \angle -57.21^\circ, E_3 = 95.5 \angle +36.19^\circ.$$

After completing the calculations, it is wise to check the sum of the voltages around the loop to see if they equal the supply E .

Whenever the rule is used to find a combination of RL or RC for either a parallel or a series circuit, it should be remembered that with some value of frequency set to a phase angle on the α or θ scale all combinations of R and L or R and C that are opposite each other will give the same phase shift. The choice of different combinations is only a difference in the impedance of the combination. For an RL combination, moving the hairline to the right will lower the impedance of the system, while for an RC combination, moving the hairline to the left yields a lower impedance. It also holds true that at any setting of the slide for all combinations of RC and RL there is a continuous relationship given between frequency and phase shift.

10.00 TRANSFER FUNCTIONS

10.10 RC Coupling, Low Frequency

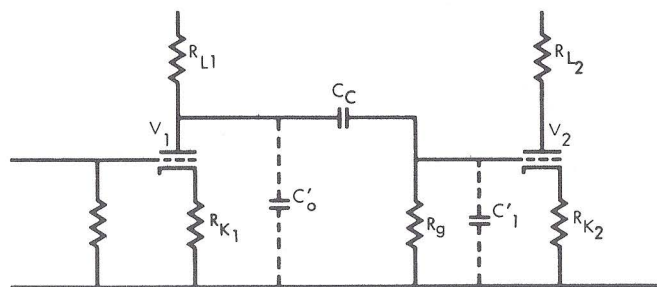


Figure 10a

The coupling between vacuum tubes to form a cascade amplifier is usually accomplished with a condenser to isolate the DC plate voltage of the driver tube, and a resistor to connect the grid of the driven tube to ground. Such a circuit is shown in Fig. 10a. The frequency response of the system is in part determined by the design used. Transformer coupling was at one time quite common but the greater flexibility of RC method has lead to its almost universal application. Certain special requirements still call for transformer or impedance coupling systems and there are many types of peaking circuits that can be found in such texts as Electronic Engineers' Handbook, by Landee, Davis & Albricht. In all of these, the N-16 will be found helpful. The solution of the RL and RC circuits play a prominent role in their solution.

In the beginning of the discussion on RC coupling it is assumed that the impedances in the plate and cathode of the tubes are pure resistance values and that the condensers are pure capacitive reactances. This is, of course, never strictly true but their deviation from the ideal is often slight and usually becomes important only at very high frequency and or very high impedances. A signal developed in plate circuit of V_1 is impressed on grid of V_2 through the coupling system consisting of C_c and R_g . The circuit of Fig. 10a, if redrawn as an equivalent network, would appear as in Fig. 10b.

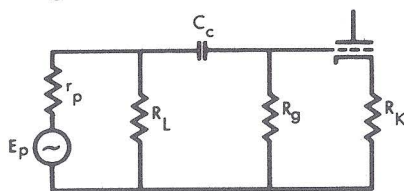


Figure 10b

E_p is the signal developed at the plate of the tube, R_p is the dynamic plate resistance, and R_L is the plate load. C_c is the coupling condenser and R_g the grid resistor of the driven tube. In the coupling network we are concerned with the transfer of the signal E_p to the grid of the driven tube. The relation can be expressed as $K = \frac{E_{g2}}{E_p}$ and the problem is to solve for the vector K .

It is assumed for the low frequency behavior that the input at the grid is an infinite impedance. This will be substantially true as long as the tube is not operated in the grid current region. The basis of the solution was discussed in Sec. 8.10, and Fig. 8a is repeated here as Fig. 10c.

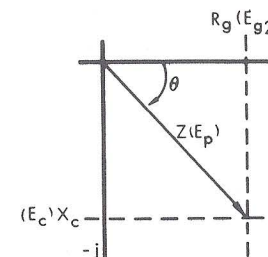


Figure 10c

Z is the series impedance of R_g and C_c and θ is the phase angle. By Ohms Law the full voltage of E_p appears across Z and $I = \frac{E_p}{Z}$. The voltage impressed on the grid of the driven tube will then be $E_{g2} = IR$. This means that in the diagram $Z = E_p$, $R_g = E_{g2}$ and X_c equals the voltage E_c developed across the condenser. From Fig. 10c,

$$\frac{R_g}{Z} = \cos \theta = \frac{E_{g2}}{E_p} = K.$$

This means that $\cos \theta$ is that portion of E_p that is impressed on the grid of the following tube. This ratio can also be expressed in decibels or db. These values are marked on the rule, $\cos \theta$ from 0.1 to 0.995, db from -0.05 to -60 and θ from 0.6° to 89.4° . A special case exists when X_c equals R_g . At this condition the phase shift is 45° , $E_{g2} = 0.707 E_p$, $Z = \frac{R_g}{0.707}$

and the power loss is -3 db, also from this point on the voltage transferred to the grid decreases by approximately 6 db for each octave of frequency change. Because of these special conditions this is considered as a reference frequency and is called the low cut-off with the symbol F_L .

A very complete discussion of amplifier coupling systems is given in Radiotron Designers' Handbook by Langford-Smith, Chapter 12 and Section 5 of Terman's Radio Engineer's Handbook.

EXAMPLES:

a. Assume that the low frequency cut-off is desired when $C_c = 0.022 \mu f$ and $R_g = 470 K\Omega$. Set the hairline to 0.47 on X_c (use dot) and move the slide until 0.22 on C scale is under the hairline. Move the hairline to the arrow and read 1.54 on F. Notice that θ is 45° , $db = -3$, and $\cos \theta$ (Relative gain) = 0.707. To find the decimal point, $0.022 \mu f = 0.22 \times 10^{-7} f$ and $470 K\Omega = 0.47 \times 10^5 \Omega$. Set -7 on C' to +6 on X_c' and at the bridge read +1. The low cut-off will then be 1.54×10 or 15.4 cycles. This setting is shown in Fig. 10d.

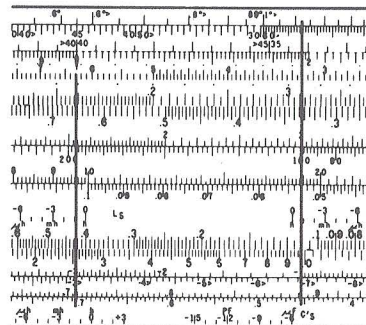


Figure 10d

b. To what low frequency will the flat response of such a coupling as in example (a) extend? Assume that anything less than 1 db is full response. With the slide set as it was for the solution of F_L in example (a), i. e., 0.22 on C to 0.47 on X_c , move the hairline to -1 on the db scale (black numerals), on F read 3 or 30 cycles, and on θ scale read the phase shift as $+27.0^\circ$.

c. If R_g is $240 K\Omega$ what value of C_c will transfer 95% of the signal at a frequency of 25 cycles? Set the hairline to 0.95 on $\cos \theta$ scale and move the slide so that 2.5 on F scale is under hairline. Shift the hairline to the dot above 0.24 on X_c and read 0.82 on C. To find the decimal point for the value of C_c , set the slide so +1 is at the bridge and below +6 on X_c' read -7 on C' . Therefore $C_c = 0.082 \mu f$.

PRACTICE PROBLEMS:

Check the answers to the following problems:

1. $C_c = 0.01 \mu f$, $R_g = 120 K\Omega$, what is the value of F_L ?
ANS. $F_L = 132$ cycles.
2. If $C_c = 0.005 \mu f$, what value of R_g will give F_L not greater than 70 cycles?
ANS. $R_g = 470 K\Omega$.

3. With $C_c = 0.022 \mu f$ and $R_g = 240 K\Omega$, what will be the phase shift and db loss at 12 cycles?

ANS. $\theta = 68.3^\circ$, Loss = -8.6 db.

4. If $750 K\Omega$ is the largest practical resistor that can be put in the grid of a particular tube, what size coupling condenser C_c will be required for the circuit to be flat within 1 db to 12 cycles?

ANS. $C_c = 0.035 \mu f$.

The factors that determine the behavior of an RC coupling network can be divided into three groups of values. These groups are:

- a. The time varying set, namely frequency F , wave length λ , angular velocity ω and the time constant τ .
- b. Any combination of R and C .
- c. The phase angle θ , the loss in db, and the relative gain $\cos \theta$.

A coupling can be evaluated by assigning both R and C which will determine its behavior or by choosing either R or C and one value from either of the other groups. The values in the unassigned group can then be found. The scales that apply to the low frequency behavior are marked with black numerals. The relationships that exist between the time varying group are:

$$\lambda = \frac{c}{F} \quad (c \text{ is the velocity of light}),$$

$$\omega = 2\pi F$$

and

$$\tau = \frac{1}{\omega} + \frac{1}{2\pi F}$$

EXAMPLE:

Find a system that will give $+56.5^\circ$ of phase shift at 0.54 Kc and have an impedance of not less than $20 K\Omega$. Set the hairline to $+56.5^\circ$ on θ and move the slide until 0.54 on F is under the hairline. All the resulting combinations of R on X_c and C_c on C will give this phase shift. However, the best set of values is yet to be chosen. By inspecting the $\cos \theta$ scale it will be found that $\cos \theta$ is about 0.55 and since $R = Z \cos \theta$, then R_g should be about $0.55 \times 20 K\Omega = 11 K\Omega$. Move the hairline to 0.11 on X_c and read 1.78 on C. Therefore $C_c = 0.0178 \mu f$.

10.20 RC Coupling, High Frequency

The high frequency behavior of coupled vacuum tubes is quite different from that at low frequency. At the high frequency limit

of their operation, the small capacitances from the tube electrodes, sockets and wiring, will all shunt the output and eventually shut off the response. The impedance of C_c is so low that it is assumed to be zero thus giving rise to the equivalent circuit shown in Fig. 10e.

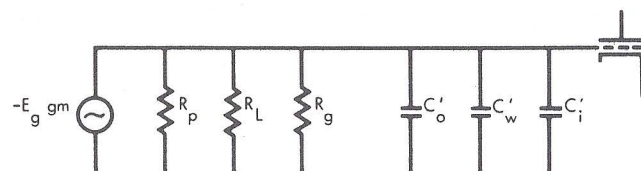


Figure 10e

Combining the resistance and capacitance values, the circuit in Fig. 10f results. R is the dynamic plate resistance, R_L is the plate load and R is the grid return of the driven tube.

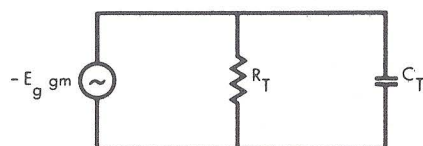


Figure 10f

If the tube considered is a triode, the value of R_p should be taken from the published characteristics for the conditions of operation. This should be accurately determined. If it is a pentode, the plate resistance is usually so much greater than the load that it can be disregarded. In most designs, R_g is large compared to R_L and, except in special cases, can be neglected, thus R_T is the parallel value of r_p , R_L and R_g . C_T is the sum of the individual values shown. C_o' is the output capacitance of the driver. For pentodes it is commonly a fixed value that can be found in a tube manual and usually has a value of 2 to 3 pf. C_o' of a triode is also a small value ranging from 0.5 to 2 pf. The exact value is found from $C_o' = C_{pk} (1 + A_k) + C_{gp} (1 + \frac{1}{A_L})$, where A_k is the gain at the cathode, and A_L is the gain at the plate, C_{pk} is the capacitance from plate to cathode and C_{gp} is the capacitance from grid to plate. All the above values are for the driving tube. C_i' is the input capacitance of the driven tube. For pentodes this again is a fixed value that can be found in a tube manual. But for a triode, this value must be calculated due to the Miller Effect. The exact value is found from,

$C_i' = C_{gk} (1 - A_k) + C_{gp} (1 + A_L)$. All the values in this equation are for the driven tube. Again A_k and A_L are the gains at the cathode and plate respectively while C_{gk} and C_{gp} being the capacitance from grid to cathode and plate respectively. C_w is the effective capacitance produced by the wiring, socket, and any other component in the system. This is usually an estimated value and with careful design can be held to 7 or 8 pf.

EXAMPLES:

a. Assume that R_T has been found to be $70 \text{ K}\Omega$ and that $C_T = 60 \text{ Pf}$. For the high frequency condition there is also a reference point where the phase shift is 45° , and the loss is -3 db which is referred to as F_H . To find F_H , set C_T on CorL to R_T on X_c and read F_H at the arrow on F scale; i. e., set 0.6 on CorL to 0.7 on X_c and at the arrow on F read 0.38. For the decimal point, $70 \text{ K}\Omega = 0.7 \times 10^5$ and $60 \text{ Pf} = 0.6 \times 10^{-10}$.

Set +5 on X_c' to -10 on C' and read +5 on F' . This gives F_H as 38 Kc as indicated in Fig. 10g.

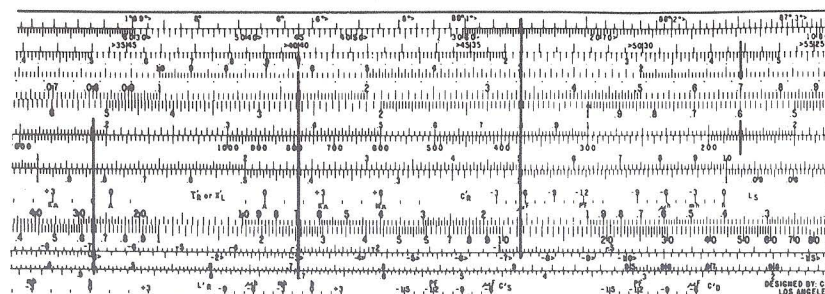


Figure 10g

Once this setting has been made, other factors for such a coupling can be determined.

b. Assume that the behavior for the conditions in example (a) were desired at 80 Kc. It will only be necessary to move the hairline to 0.8 on F . On the α scale read -64.6° for the phase shift (red numerals) and a coupling loss of -7.4 db (red numerals). Find the frequency at which the loss is -1 db? Move the hairline to -1 on the db scale (red numerals) and read 19.2 Kc on F . These headings are also shown in Fig. 10g.

c. To demonstrate the flexibility of the rule, assume that it is required that a system reach 24 Kc with no more than 1 db of loss. Leave the hairline set to -1 db (from the previous example) and move the slide until 0.24 on F is under the hairline.

0.48 on CorL is now against 0.7 on X_c . This means that the value of C_T must be reduced to 48 Pf to reach the desired frequency. This is a large change and hard to achieve. Assuming that C_T cannot be reduced, then at 0.6 on CorL is 0.56 on X_c . By reducing R_T the response will be obtained. If a triode is the driver, this means reducing the plate resistance which usually controls the value of R_T . This is done by operating at higher plate current with a possible loss in gain. If a pentode is being used, the operation necessitates a reduction of R_L with a decrease in gain. For those that wish to calculate the exact solution of a triode coupling where allowance is made for the degenerative effect of an impedance in the cathode, the following information might prove helpful.

The gain at the plate of a triode is given by:

$$A_L = \mu \frac{Z_L}{Z_L + R_p + Z_k (1 + \mu)}$$

while the gain at the cathode is given by:

$$A_k = \frac{Z_k}{Z_L + R_p + Z_k (1 + \mu)}$$

In both of these equations the values Z_L and Z_k are complex and the equations must be solved for the complex values of A_L and A_k . From the results of A_L and A_k of the driven tube, the correct value of C_i can be found and this becomes part of Z_L of the driver. Bear in mind that Z_L is the complete complex impedance between the plate of the driving tube and ground.

PRACTICE PROBLEMS:

In the following practice problems, thought must be given to the discussion above on the behavior of high frequency coupling circuits. A certain amount of analysis is required to get the answers.

1. A pentode is to be used to drive another tube that has a C_i of 6.5 Pf, $C_w = 6$ Pf and the C_o of the driving pentode is 3 Pf. What must be the value of R_L to have no more than 0.5 db at 750 Kc?

ANS. 4.95 K Ω .

2. A pair of triodes are to be connected in cascade with the cathode resistors bypassed. R_T is calculated to be 7K Ω with $C_o = 3$ Pf and $C_w = 8$ Pf. The driven tube has a $C_{gp} = 1.5$ Pf and $C_{gk} = 1.6$ Pf. How much gain can be designed into the driven tube so that F_H will be at least 750 Kc?

ANS. A gain of 10.0.

3. A pentode is to be used to drive the deflection plates of a cathode ray tube. C_o of the driver is 2.3 Pf, the deflection plates have a capacitance of 12 Pf and 5 Pf must be allowed for C_w . The requirement is that there must be no more than 20° phase shift at 2.5 Mc. What is the value of R_L that can be used in the pentode and what will be the gain if $g_m = 7000$ micromhos?

ANS. $R_L = 1000 \Omega$, $A_L = 7$.

10.30 Transfer Function, General

There are four types of circuits that can be solved directly on the rule as in the previous examples. These are shown in Fig. 10h.

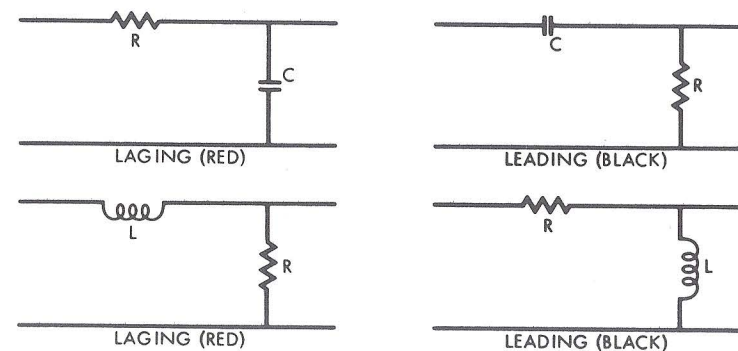


Figure 10h

In these circuits the terms "Lead" and "Lag" are referring to the phase relation between the input and output voltages. The usual convention is to use these terms for the relationship between the applied voltage and the current. The definitions used here help in deciding on the proper scale to read. For the analysis of any of the above networks, the value of capacitance or inductance on the L or C scale is placed opposite the value of R . If a RC network is involved the value of R is read on the X_c scale and on the X_L scale if the network is RL. If the circuit is a "LEADING" one, the phase angle is read on the θ scale and its sign is positive. If the circuit is a "LAGGING" one, the phase angle is read on the α scale (red numerals) and its sign is negative. Another way that is helpful in remembering just which scale is read is that: when the output increases with increasing frequency the circuit is a leading one by the definition above and all readings are made on the black scales (excepting the X_L which is used for all RL networks). When the output decreases with increasing frequency it is a lagging circuit and readings are

made on the red scales. For those familiar with the "s" plane type of analysis, a system of the type $\frac{s}{s + 1/\tau}$ is solved on the black numbered scales and $\frac{1}{s + 1/\tau}$ is read on the red numbered scales.

The technique described above is also used in transistor circuits. The method of finding the driver and driven impedance factors is, however, much more complicated. The subject is too lengthy for a manual of this type. However, when one has found the impedance factors, the coupling circuit can be solved in the same manner as for vacuum tubes.

11.00 FEED BACK SYSTEMS

In systems that have the general configuration shown in Fig. 11a, the N-16 offers a means of rapid analysis.

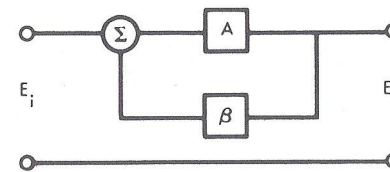


Figure 11a

There are many mathematical techniques that have been developed to analyze the stability of this type of system. One of the earliest of these is the "Nyquist" diagram. The limitation of this method lies in the fact that it is not analytic, i. e., one cannot assign the conditions desired and then solve for the correct constants that will achieve this result. The method requires that the circuit constants be assigned and then analyzed to determine the results. Much of the difficulty in feedback systems comes from the instability that can result even when it is being applied to increase the system stability. Some systems are deliberately designed to achieve this instability. An oscillator is an unstable feedback amplifier, the instability is the aim and purpose of the design. In solving the Nyquist diagram it is required that many points on the frequency spectrum be worked out by complex algebra and plotted. This is not only laborious but also time consuming with the results occasionally left for experimental evaluation. The experimental method is, of course, the final criterion but rather inflexible. For those that are not already familiar with this type of analysis, a brief outline with a sample problem follows. The subject of feedback is a major topic in all forms of engineering and life sciences. So the discussion here will be minimal. ⁽¹⁾

The basic equation of feedback systems is:

$$A' = \frac{A}{1 - A\beta} \quad A' = \frac{E_i}{E_o}$$

- (1) For further study, the Nyquist system is covered well in "Radiotron Designer's Handbook" by Langford-Smith, Stability Criterion in "Handbook of Automation, Computation and Control," Vol 1 by Grabbe, Ramo, and Wooldridge.

where

A = complex gain of the amplifier

β = portion of E_o that is returned to the summing circuit, it is complex.

The problem of determining stability by the Nyquist criterion is the solution of $A\beta$ at the critical points of the frequency spectra. In a conventional amplifier, over the midband region A is positive while β is negative and $A\beta$ therefore is negative. Therefore $1-A\beta$ is greater than unity. This makes A' less than A . If at any time $A\beta$ becomes a positive number and less than unity, the denominator becomes less than unity and A' becomes greater than A . When and if $A\beta$ becomes +1 then the denominator is zero and A' becomes infinite. An amplifier with infinite gain is one that has an output with no input signal required.

$$A' = \frac{E_o}{E} = \frac{E_o}{0} = \infty$$

This is an oscillator, an output with no input. The Nyquist plot is a study of the behavior of a system where $A\beta$ becomes complex and in particular where $(1-A\beta)$ is less than unity. Fig. 11b is a polar plot of the locus of the vector $A\beta$.

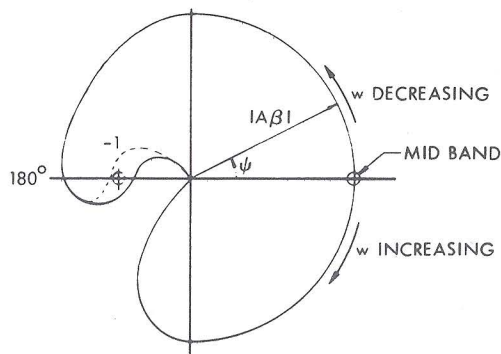


Figure 11b

This plot is the curve traced by the end of a line whose length is the magnitude of $A\beta$ and pointing in a direction equal to the sum of the phase angles A and β . Most amplifiers have a flat response over a broad range of frequencies called the "Mid Band" where the phase angle is substantially zero. This region is marked in the diagram. As the reactances of the systems in both A and β begin to have effect either at low or high frequency, the product $A\beta$ changes in value and the sum of their phase angles develops an appreciable magnitude. It could then be determined whether the system is stable or not by making

a polar plot. If such a plot does not encircle the point -1 (a magnitude of 1 and 180° angle), the system is stable. If the plot does encircle this point, the system will be unstable and will oscillate.

The conventional way of making the plot in Fig. 11b is to use +1 as the critical point since instability occurs when $A\beta = +1$. Here, however, we are making a polar plot and it is simpler to measure angles from the plus axis. Included in this manual is a plotting sheet of the region close to the -1 point. The purpose of this sheet will become apparent with further discussion of the use of the rule in feedback calculations. In Fig. 11b, the system represented will be stable at high frequencies but will oscillate at some low value. If the low frequency trace had crossed the abscissa the second time as shown by the dotted line the system would have been quasi-stable, i. e., it would be stable just as long as the gain of the amplifier did not reduce and bring the line across the -1 point. This latter effect is found in some pieces of equipment that will operate in a satisfactory manner when new; but as the system ages and the gain falls off, and instability occurs.

EXAMPLES:

a. As a sample problem, the response of a simple two stage amplifier will be used. The circuit to be considered is shown in Fig. 11c.

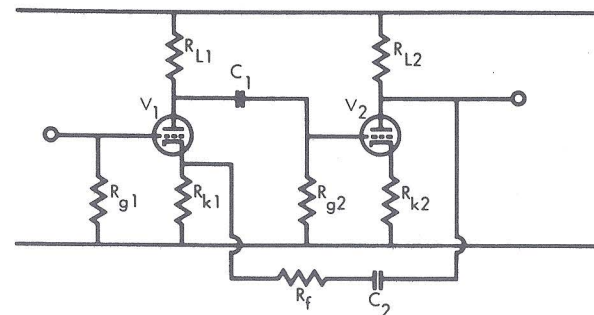


Figure 11c

It will be assumed that the gain of V_1 , $A_1 = 40$ and of V_2 , $A_2 = 25$, so that $A_o = 1000$. It will further be assumed that the value of R_{L2} is low enough so that the impedance of the feedback network of R_f & C_2 will not materially alter the plate loading of V_2 .

The behavior of an amplifying stage may be expressed as the product of the tube gain times the loss factor of the coupling network that feeds the signal to the next stage or output point, thus,

$A = |A|K$. Here A is the product of the scalar value $|A|$ (since all the reactive values are assumed lumped in the coupling system the tube gain is only a magnitude) and the vector K . This method can be carried out completely around the loop of a system whether closed or open. If the scalar value is an amplifier, its value will be greater than unity and if a loss, less than unity. For the circuit in Fig. 11c as we start at the grid of V_1 , through V_2 and then by the way of C_2 and R_f to the cathode of V_1 the two values of gain are $A_0 = 1000$. β_0 is determined by R_f and R_{K1} and the two vector values of the coupling systems, C_1 , R_{C2} and C_2 , R_f . For the sake of simplicity call the coupling systems K_1 and K_2 . From this the gain $A = A_0 K_1 / \theta_1$ and $\beta = \beta_0 K_2 / \theta_2$ and $AB = A_0 \beta_0 K_1 K_2 / (\theta_1 + \theta_2)$. Using this notation we can write the feedback equation:

$$A' = \frac{A_0 K_1 / \theta_1}{1 - (A_0 \beta_0 K_1 K_2 / (\theta_1 + \theta_2))}$$

In the amplifier of the problem, assume that A' is to be 50, then since

$$A_0 \beta_0 = \frac{A_0}{A'} - 1 = \frac{1000}{50} - 1 = 19 = +12.8 \text{ db and } \beta_0 = \frac{19}{1000} = 0.019$$

Assume that $R_{K1} = 3.3 \text{ K}\Omega$, then since

$$\beta_0 = \frac{R_{K1}}{R_{K1} + R_f}, \text{ we have } R_f = R_{K1} \frac{1 - \beta_0}{\beta_0} = 3.3 \times 51.6 = 170 \text{ K}\Omega.$$

By using decibels in the solution, products may be found by simple addition. Since the rule is calibrated in power decibels, i. e., $\text{db} = 10 \text{ Log}_{10} \frac{P_2}{P_1}$ these values will be used instead of the voltage value where

$$\text{db} = 20 \text{ Log}_{10} \frac{V_2}{V_1}.$$

The results will be the same as long as it is kept in mind that these db values simply represent a ratio. To show the effects of a feedback system that is not well designed, the time constants of the coupling circuits will be chosen closer together than good practice prescribes. Let $C_1 = 0.0022 \text{ }\mu\text{f}$, $R_{C1} = 1 \text{ meg}$. This will give $\tau_1 = 0.0022 \text{ sec} = 2.2 \text{ ms}$. Let $\tau_2 = 0.005 \text{ sec} = 5 \text{ ms}$. The time constant can be read from the rule in two different ways.

RULE: If the value of the time constant τ is set to the arrow, then the values of RC and RL will be opposite each other

on their respective scales, i. e., L and C on the LorC scale, R for an RL circuit on X_L , and R for an RC circuit on the X_C scale. Whatever value of τ is set to the arrow, the same value will be at the small arrow in the middle of CorL on the X_C scale. Both of these methods may be used.

Set $\tau = 0.22$ to the arrow. This will give a choice of any combination of R and C for the coupling K_1 . As mentioned earlier, $R = 1 \text{ meg}$ and $C = 0.022 \text{ }\mu\text{f}$. This is found to be correct since 1 on C is at 0.22 on X_C . To find the correct values for K_2 , set 0.5 on τ to the arrow (for 5 ms), move the hairline to 0.17 on X_C (since R_f is found to be $170 \text{ K}\Omega$), and read 2.95 on C. After locating the decimal point, this value will be $0.0295 \text{ }\mu\text{f}$. $0.03 \text{ }\mu\text{f}$ could be used without introducing any error. Above each of these τ values could be read F_L on F scale. $F_{L1} = 72$, and $F_{L2} = 31.8$. To complete the analysis of the system, it is only necessary to make a table of the coupling loss and phase shifts at various frequencies. This is done according to the discussion in sections 10.10 and 10.20. Fig. 11d shows the setting of finding K_1 , with the first few readings indicated.

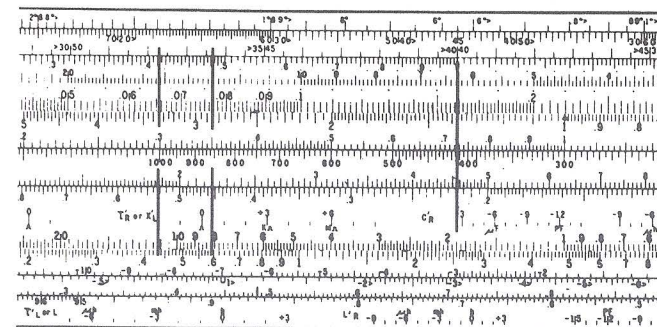


Figure 11d

The readings to be taken for each frequency point are the phase angle θ or α (depending on whether one is studying the stability at the low or high end of the spectrum), and the loss in db. For the example under discussion, the stability at the low end will be studied. Notice that in Fig. 11d, 0.22 on τ is set to the arrow and 0.22 on CorL is at 1 on X_C for $0.0022 \text{ }\mu\text{f}$ and 1 meg respectively. Also at the arrow on F is 0.72 for a value of $F_L = 72$ cycles. Move the indicator to 0.35 on F for 35 cycles and read -7.2 db (black numerals) and $+64.2^\circ$ on θ (black numerals). Thus at 35 cycles $K_1 = -7.2 / +64.2^\circ$. Following the same procedure for 30 cycles gives $K_1 = -8.3 / +67.5^\circ$. By this process

the table in Fig. 11e was constructed. Values for K_2 are obtained by setting the value τ_2 to the arrow and reading the values of db loss and phase angle at this setting for the selected frequencies.

F	K_1	θ_1	K_2	θ_2	$K_1 K_2$	$\theta_1 + \theta_2$	$A\beta$
40	-6.3 db	61.0°	-2.1 db	38.5°	-8.4 db	99.5°	+4.4 db
35	-7.2	64.2	-2.6	42.3	-9.8	106.5	+3.0
30	-8.3	67.5	-3.2	46.6	-11.5	114.1	+1.3
28	-8.8	68.8	-3.6	48.6	-12.4	117.4	+0.4
26	-9.4	70.2	-4.0	50.6	-13.4	120.8	-0.6
24	-10.0	71.6	-4.4	53.0	-14.4	124.6	-1.6
22	-10.7	73.1	-4.9	55.4	-15.6	128.5	-2.8
20	-11.5	74.6	-5.5	58.0	-17.0	132.5	-4.2
18	-12.3	76.0	-6.1	60.5	-18.4	136.5	-5.6
15	-13.8	78.3	-7.4	64.8	-21.2	143.1	-8.4
10	-17.3	82.2	-10.4	72.6	-27.7	154.8	-15.0

Figure 11e

The columns for K_1 , θ_1 , K_2 , and θ_2 are read directly from the rule. Since all K values are read in db, the product $K_1 \times K_2$. Included in this manual is a polar chart on which the absolute value of $A\beta$ can be plotted. The most convenient way of handling this is to slip the chart under a sheet of transparent paper and plot the points found in the chart. This plot can be used to find the degree of stability of the system and the behavior of the final design. There is a series of concentric black circles marked in db and radial lines marked in degrees. The value of $A\beta$ are marked on the tracing paper and joined with a smooth line. The amplifier will not oscillate if the trace thus formed does not enclose the -1 point. There is also a series of red circles about the -1 point. These are marked in db, with a few radial lines marked in degrees. These red graduations are the value of $(1 - A\beta)$ and can be used to find the behavior of the amplifier. Where the plot crosses the red 0 db circle, the amplifier is unaffected by feedback. Where the plot is outside this circle the effect of feedback is negative while inside it is positive. For the problem on hand, this transition takes place between 26 and 28 cycles. The equation to be solved is:

$$A' = \frac{A}{1 - A\beta} = \frac{A_0 K_1 / \theta_1}{1 - A_0 \beta_0 K_1 K_2 / \theta_1 + \theta_2}$$

The A' value in db is gotten by subtracting $(1 - A\beta)$ in db from A in db. All of these must be solved for point by point. In the plot of the example, notice that the line comes closest to the -1 point at about 26 cycles. As a check for those that wish to carry out the process, some of the values are given in Fig. 11f.

F	$1 - A\beta$	ψ	$A_0 K_1$	θ_1	A' (db)	θ'	GAIN
MB	-13.0 db	0°	+30.0 db	0°	+17.0	0°	50.0
28	-0.4	63	+21.2	68.8	+20.8	+5.8°	120.0
26	+0.3	53	+20.6	70.2	+20.9	+17.2	123.0
24	+0.7	42	+20.0	71.6	+20.7	+29.6	118.0
22	+1.0	33	+19.3	73.1	+20.3	+40.1	107.0
20	+1.0	25	+18.5	74.6	+19.5	+49.6	89.0
18	+1.0	20	+17.3	76.0	+18.3	+56.0	67.6

Figure 11f

NOTE: In the chart above, the gain has increased above the midband (MB) of 50.0 and reached a maximum of 123.0 at 26 cycles. This effect is due to having the time constants of the two networks K_1 and K_2 too close together. To see the effect of this, work out a solution having $\tau_2 = 20 \tau_1$.

12.00 DISTRIBUTED SYSTEMS

12.10 Transmission Lines

In their rigorous form, transmission lines have a rather complex mathematical structure. Most of the time, however certain approximations can be made that will give answers well within engineering requirements.

The impedance of a line to a suddenly applied signal is given by:

$$Z_o^2 = \frac{R + j\omega L}{G + j\omega C} \quad \text{where} \quad \begin{array}{l} R = \text{Series resistance} \\ L = \text{Series inductance} \\ G = \text{Shunt conductance} \\ C = \text{Shunt capacitance} \end{array}$$

All values are for unit length.

In many cases the value R is small compared to $j\omega L$ while G is much larger than $j\omega C$. When this case exists, the above equation reduces to $Z_o^2 = \frac{L}{C}$. This new relationship can be solved directly on the N-16. In Fig. 12a is a schematic sketch of a unit length of a transmission line.

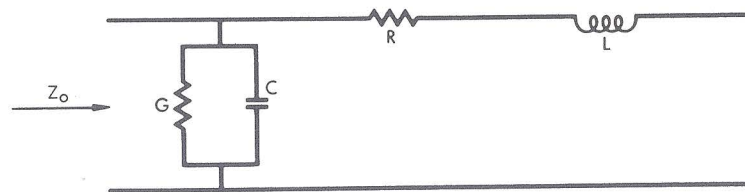


Figure 12a

EXAMPLE:

a) Assume that the series losses (R) and the shunt leakage (G) are negligible, which is usually the case. Let $L = 0.075 \mu\text{h}$ and $C = 30 \text{ Pf}$. What is the value of Z_o ?

(The rule is marked in Z_s which is sometimes used for the same factor namely "Surge Impedance". Z_o appears in many text books but the meaning is the same). To 0.75 on L_r set the arrow of the left index of C_r ; move the hairline to 3.0 on C_r and read 0.50 on Z_s . This setting is shown in Fig. 12b.

For the decimal point:

$$\begin{aligned} 0.075 \mu\text{h} &= 0.75 \times 10^{-7} \text{ H} \\ \text{and} \quad 30 \text{ Pf} &= 3.0 \times 10^{-11} \text{ f.} \end{aligned}$$

so set -7 on L_s to -11 on C_s and read +2 at the bridge on F' . Therefore $0.50 \times 10^2 = 50 \Omega$ and $Z_o = 50 \Omega$.

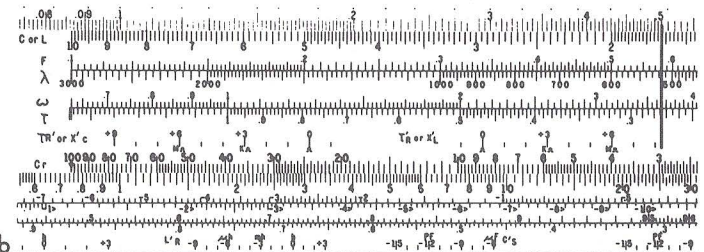


Figure 12b

The above calculation shows that a $0.075 \mu\text{h}$ and 30 Pf per unit length (often given as per meter) transmission line, will appear as a 50Ω resistance to signals applied to the input until that signal has had time to reach the end of the line and be reflected to the input. If the end of the line has a 50Ω resistance across the terminals then the transmission line will appear as 50Ω for any type of signal applied. When the end of the line is terminated in any value other than its characteristic impedance Z_o , the input can appear to be almost any combination of resistance and reactance depending on the character of the termination. The general analysis of transmission line behavior involves the use of hyperbolic functions marked SH1, SH2, and TH which are on the math side of the rule. An excellent explanation of hyperbolic functions as applicable to a slide rule is given in the Pickett manual for Dual Base Log Log Slide Rules.

PRACTICE PROBLEMS:

1. The series inductance of a pair of parallel wires is 3.36 mh/mile and the shunt capacitance is $0.0085 \mu\text{f/mile}$. What is the surge impedance Z_o ?

ANSWER: 630Ω .

2. A microphone cable has a measured capacitance of 217 Pf and an inductance of $0.165 \mu\text{h/foot}$. If the cable is 7 feet long, what will be its characteristic impedance?

ANSWER: 75Ω .

3. A micro-strip has a series inductance of $0.0074 \mu\text{h}/\text{foot}$, what will the capacitance per foot have to be to bring the value Z_0 down to 3Ω ?
ANSWER: $820 \text{ Pf}/\text{foot}$.

12.20 Delay Time

An electromagnetic signal does not travel down a transmission line at the same velocity it does in a vacuum or air. The velocity in air is very close to that in a vacuum. Such a signal is delayed, i.e., it travels more slowly. The amount that the transmission signal lags behind the air signal is called the delay time. This is given by

(1) $T_D = Z_0 C$ where T_D is the delay per unit length, Z_0 is the characteristic impedance, and C is the capacitance per unit length. If $Z_0^2 = \frac{L}{C}$ is substituted, the above equation becomes

(2) $T_D = \sqrt{LC}$ or $T_D^2 = LC$
which can be solved on the N-16 by a single setting. (Note: $Z_0 C$ in equation (1) is a resistance value times a capacitance which, as we have seen, is a time value or time constant.)

EXAMPLE:

Using the same values as in practice problem 1 of section 12.10 (i.e., $C = 0.0085 \mu\text{f}$ and $L = 3.36 \text{ mh}$) solve for T_D . Set 8.5 on Cr to 3.36 on Lr and at the small arrow on CorL read 0.532. Checking for the decimal point, we find that L has a magnitude of -3 and C has -9. Since the sum of these two magnitudes is an even number, the setting is satisfactory. To -9 on C_D' set -3 on L_D and at the bridge read -5 on τ' . Then the delay is $5.32 \mu\text{s}$ per mile.

PRACTICE PROBLEMS:

- Find T_D for practice problem 2 of section 12.10.
ANS. $T_D = 0.0226 \times 10^{-7} = 0.00226 \mu\text{s} = 2.26 \text{ ns}$.
- Find T_D for practice problem 3 of section 12.10.
ANS. $T_D = 0.246 \times 10^{-8} = 2.46 \text{ ns}$.

13.00 DYNAMIC ANALOGIES

There are many types of problems, other than those encountered in electronics, that can be calculated on the N-16. The method of "Dynamic Analogies" converts mechanical (both rectilinear and rotational), acoustic and hydraulic systems to their equivalent electrical networks. Once the equivalent electrical network has been established, the system is solved as an electrical circuit. This method is a powerful tool in the hands of those familiar with the conversion technique. For a detailed discussion, see the Handbook of Automation, Computation, and Control, Vol. 1, Sec. 20, by Grabbe, Ramo, and Woolridge. Also, Dynamic Analogies by Olson. The scales of the N-16 are arranged to solve equations of the basic type such as $X_C = \frac{1}{\omega C}$, $X_L = \omega L$, and

$\omega^2 = \frac{1}{LC}$. The numerical calibration involves no constants so that the rule reads the numerical value of the solution. This means that any kind of system arranged in any of the above forms can be solved on the rule in the same manner as an electrical system in accordance with the theories covered above. To apply Newton's First law $F = ma$ (Where F = force, m = Mass, and a = acceleration), we have to make the analogies that $F = X_L$, $m = L$ and $a = \omega$. Any system of consistent units can be used and the decimal point can be found in the same manner as for the equation $X_L = \omega L$.

EXAMPLE:

Using the GCS system, what will be the acceleration "a" of a 15 gm mass acted upon by a force of 1750 dyne? Set 1.5 on L to 17.5 on X_L and at the arrow read 11.65 on ω . To find the decimal point, set +2 on X_L' to +1 on L' and at the bridge read +1. Therefore $a = 11.65 \times 10 = 116.5 \text{ cm}/\text{sec}^2$.

This is of course a very simple problem that might just as well be solved on the C and D scales of the math side but it helps clarify the principle involved.

The electronic scales become more useful when calculating problems dealing with mechanical vibration. The resonant frequency of a simple system of a mass suspended on a spring is given by $\omega^2 = \frac{K}{m}$. Where ω is angular velocity of a rotating vector in radians per second, m is the mass of the vibrating system, and K is the stiffness of the spring in units of force/

displacement, such as

$$\frac{\text{pounds}}{\text{inches}} \quad \text{or} \quad \frac{\text{dynes}}{\text{cm}} .$$

If the spring constant is changed to compliance which is equal to $\frac{1}{K} = C$ or

$$\frac{\text{Displacement}}{\text{force}} = \frac{\text{inches}}{\text{pounds}} = \frac{\text{cm}}{\text{dynes}}$$

etc., the vibration equation becomes: $\omega^2 = \frac{1}{mc}$

which has the same form as the electrical circuit where $\omega^2 =$

$\frac{1}{LC}$. In mechanical systems, mass is equivalent to inductance and C (compliance value) is equivalent to capacitance.

EXAMPLES:

a) What would be the resonant frequency of 1.5 lb weight suspended on a spring that deflects 0.5 inches due to this weight?

$$C = \frac{\text{displacement}}{\text{force}} = \frac{0.5 \text{ in}}{1.5 \text{ lb}} = 0.333 \text{ in/lb}$$

Mass

$$m = \frac{\text{Weight}}{g} \quad \text{where, } g = \text{Acceleration due to gravity}$$

$$g = 386.4 \text{ in/sec}^2$$

$$m = \frac{1.5}{386.4} = 3.88 \times 10^{-3}$$

To find F or ω , set 3.33 on Cr to 3.88 on Lr and at the arrow read F = 0.444 and $\omega = 2.79$.

To locate the decimal point note that $0.333 = 3.33 \times 10^{-1}$ and $m = 3.88 \times 10^{-3}$, the sum of the magnitudes is -4. Set -3 on C'r to -1 on L'r and at the bridge read +1. Therefore F = 4.44 cycles and $\omega = 27.9 \text{ rad/sec}$.

b) Assume that the moving system of a phonograph cartridge has a mass of 15 mg and a compliance of $3.5 \times 10^{-8} \text{ cm/dyne}$. What will be its resonant frequency?

Set 3.5 on Cr to 1.5 on Lr and at the arrow read 0.698 on F. To locate the decimal point remember that the unit of mass is the gram, hence $15 \text{ mg} = 1.5 \times 10^{-2} \text{ gm}$. Therefore, to find the magnitude of F set -8 on C'r to -2 on L'r and at the bridge read +4. $F = 0.698 \times 10^4 = 6980 \text{ cycles or } 6.98 \text{ Kc}$.

To cover all the possible uses of the N-16 in connection with "Dynamic Analogies" would require a full sized textbook and therefore cannot be included in a slide rule manual of this type. However, the types of equations that the electronic scales will handle are listed below for easy reference. Any subject under discussion that has an expression of the same type can be handled on the rule by an appropriate assignment of terms.

$$X_L = \omega L = 2\pi FL \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi FC}$$

For both of the above relationships, the reciprocal of F can be read on the CorL scale which is the time of one cycle.

$$\alpha = \text{Arctangent} \frac{\omega L}{R} \quad \theta = \text{Arctangent} \frac{X_C}{R} = \frac{1}{\omega RC} = \frac{1}{\omega \tau}$$

In the equation above, ω can be replaced by $2\pi F$.

The tangent of θ is read on the D or Q scale.

$$\omega^2 = \frac{1}{LC} \quad Z_o^2 = \frac{L}{C} \quad T_o^2 = LC$$

In these relationships ω^2 can be replaced by $4\pi^2 F^2$.

$$\tau = RC \text{ and also, } \tau = \frac{L}{R} .$$

14.00 SUMMATION

RC Coupling Network

The interstage RC coupling network commonly used between amplifier stages can be calculated in a single setting on the N-16. This is accomplished by a series of scales that are arranged to show the relationship between a real component (R) and an imaginary component ($\pm jX$). The scales will then read the vector angle θ for negative values in the fourth quadrant, and α for positive values in the first quadrant. The series of scales also include the tangent and cosine scales of θ and α as well as the relative power ratio of the transfer in db.

Low Frequency Response

The low frequency behavior of an RC coupling system is usually a circuit having the form

$$\frac{S}{S + \frac{1}{\tau}}$$

To calculate this value, the rule may be set in any of several ways. The RC combination may be assigned, the time constant τ or F_L expressed in frequency, and regular velocity or wave length may be used. For any value of the time constant (set to the arrow in the middle of the D scale) at the arrow can be read the frequency F, the wavelength λ , and the angular velocity ω of the -3 db cutoff point.

At the same setting, the capacitance values on the CorL scale combined with the values of R as read on X_C will all have this time constant and will give the same coupling response. Leaving the slide as set and moving the hairline to any other value of the time functions, the phase angle may be read on the θ scale, the $\tan \theta$ on the D scale, $\cos \theta$ on the cos scale, and the db loss on the db scale. All the readings for a lead circuit are read on the black numbered scales. The db and θ scales are extended on the upper edge of the rule to read up to -60 db and 0.6° by changing the time function scales by two decades. The color designation still applies.

High Frequency Response

The high frequency behavior of a coupling network can usually be represented by the lag circuit

$$\frac{1}{S + \frac{1}{\tau}}$$

This type of system can also be calculated on the N-16 in the same manner as the lead type discussed above. The only difference is that readings are made on the scales with red numerals. The lag angle is marked α on the rule and there are db and $\cos \theta$ scales for this type of solution.

The X_L scale is calibrated in the same direction as the CorL scales and the X_C is inverted so that this combination will facilitate multiplication or division. This will be found convenient in finding (Z) once $\cos \alpha$ or $\cos \theta$ is determined.

The resonant frequency scales C_R and L_R are designed to solve three types of equations.

$$(1) \quad \omega^2 = \frac{1}{LC} \quad (2) \quad R^2 = \frac{L}{C} \quad (3) \quad \tau^2 = LC$$

RULES:

These three relationships represent the undamped resonant frequency of a series or parallel LC system, the characteristic impedance and delay time of a distributed line. For equation (1), at any setting of the slide, the values of C on C_R and L on L_R represent combinations of L and C that will be resonant at the value of ω read at the arrow. For equation (2), set the arrow at the left index of C_R to the value of L and move the hairline to C on C_R and read the characteristic impedance Z_s on the Z_s or X_C scale. For equation (3) set L on L_R to C on C_R and at the arrow in the middle of the CorL scale read T_0 . This scale is marked τ at the right end.

Time Constants

RULE: Time constants of RC and RL circuits can be read by setting C on the CorL scale to R on the X_C scale and reading τ on the X_C scale at the arrow in the middle of the CorL scale or at the main arrow on the τ scale. The same method is used for RL by setting L on the CorL scale and reading τ as for CR combinations. τ may also be read on the X'_C scale at the arrow in the middle of the CorL scale.

The Decimal Point Location

Located along the bottom of the rule and on the slide are sets of scales marked with a prime ('). These scales are used to find the decimal point applicable in the operation of the rule.

The rule is calibrated in basic units such as cycles, ohms, farads, and henrys, — etc. The primed scales compute the power of ten (magnitude), that is to be applied to the answer on the basis of the units used. At the right side of the slide is a scale marked F' and T' . This scale is read at the inner edge of the metal bridge (called the "bridge" in the text). This scale reads the magnitude of the time functions F , λ , ω , and τ . The magnitude for ω is the same as that for F . λ is calibrated in unit values times 10^5 and allowing for this, its magnitude is read on the F scale. The method of use is to find the power of 10 if applied to the value as read on the rule, that will express the problem value in basic units.

EXAMPLE:

$0.0033 \mu f = 0.0033 \times 10^{-6} f = 0.33 \times 10^{-8} f$ where 0.33 is the calibration point read on the rule. If, instead of setting the rule to 0.33 the reading was made at 3.3 on the CorL scale then $0.0033 \mu f = 3.3 \times 10^{-9}$. Assume we want to know the value of X_c of this condenser at 1.8 Kc. To the arrow set 1.8 on F and at 0.33 on C read 0.268 on X_c' . Since $0.0033 \mu f = 0.33 \times 10^{-8}$ and $1.8 \text{ Kc} = 1.8 \times 10^3 \text{ cycles}$ the magnitudes are -8 for C and $+3$ for F . To find the magnitude of X_c' set $+3$ (Kc) to the bridge and the hairline to -8 on C' , on X_c read $+5$. Then $0.268 \times 10^5 = 26.8 \text{ K}\Omega = X_c'$.

The following table shows the scales used for the circuit values and the solution desired:

CIRCUIT VALUES	SCALES
C , X_c and $F(T)$	C' , X_c' and F'
C , R and τ	τ_c' , τ_r' and τ'
L , X_L and $F(T)$	L' , X_L' and F'
L , R and τ	τ_L' , τ_r' and τ'
L , C and $F(T)$	L_r' , C_r' and F'
L , C and Z_s	L_s' , C_s' and F'
L , C and τ	L_D' , C_D' and τ'

PRACTICE PROBLEMS AND ANSWERS