THE EXTRA SECTIONS OF THE LOG LOG SCALE

Model 803 provides an extra section of Log Log scales. This section is used in exactly the same way as the rest of the LL scales (LL1, LL2, LL3) and requires no additional explanation. Note that it is possible to set numbers near 1 to great accuracy; thus 1.00333, which is a six figure number, is easily set. An example which illustrates the added convenience of having this scale is given below.

Example 1. Find $1.0261^{0.342}$. Set the left index of the C scale opposite 1.0261 of LL1+. Move the hairline over 342 of C, and read the result on the LL0+ scale as 1.00886.

Note that if the LL0+ scale were not on the slide rule, the *answer could not be read directly*. In that case (that is, with slide rules in which the lower bound of the range of the LL scales is 1.01 instead of 1.001), the result may be computed by using logarithms, but with less accuracy.

Thus, set the hairline over 1.0261 of the LL1+ scale, and the left index of the C scale under the hairline. The reading on the D scale gives $\log_e 1.0261$, which is 0.0258. Move the hairline over 0.342 of C (which multiplies 0.0258 by 0.342), and read 0.0088 on the D scale. This is $\log_e 1.261^{0.342}$. Now $\log_e (1 + X) = X$, approximately, if X is sufficiently small. In this example, let $X = 0.0088$ (which is small), and then the number, or $1 + X$, is 1.0088.

With Model 803 it is unnecessary to resort to this longer procedure. Moreover, by using the DF_M scale and base 10, it is possible to observe how the work described above is automatically done. Thus, when the hairline is over 1.0261 of LL1+, then $\log_{10} 1.0261 = 0.0\ 1119$ is on the DF_M scale under the hairline. If this is multiplied by 0.342 using the C or CI scales, the result is 0.00383 on DF_M. In other words, $\log_{10} 1.0261^{0.342} = 0.342 \log_{10} 1.0261$. The number corresponding to this logarithm is, of course, 1.00886, read on LL0+.

The readings on the LL0– scale are reciprocals of the readings on the LL0+ scale. This scale extends the LL1– scale from 0.99 to 0.999 which is, of course, much closer to 1. This scale is used in the same way that the LL1–, LL2–, and LL3 scales are used. No additional explanation beyond that given in the manual for these scales should be needed.

PRINTED IN USA

FORM M-14S

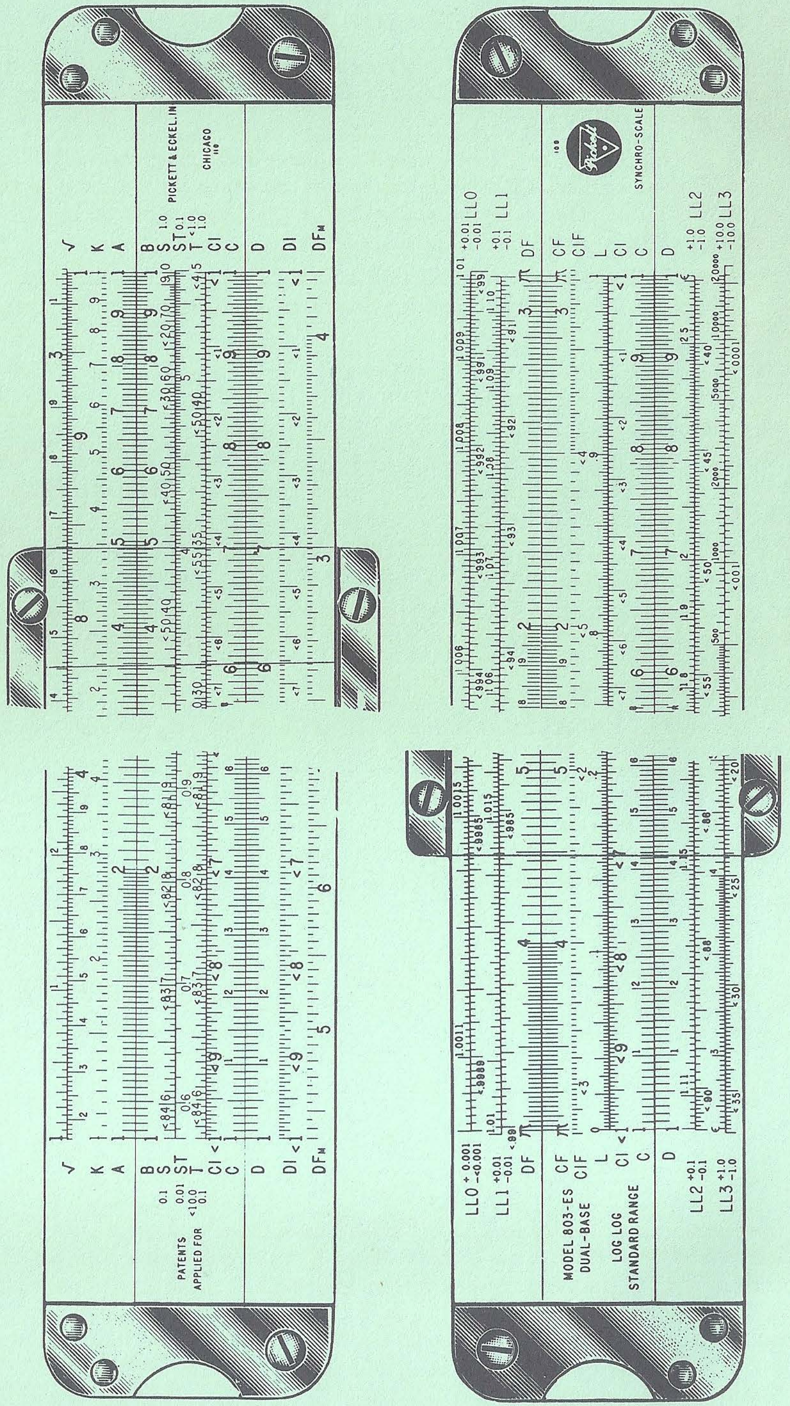
www.reglasdecalculo.com

Supplement to Log Log Manual M-14...

how to use the MODEL 803 LOG LOG Dual Base SPEED RULE

by **MAURICE L. HARTUNG**
Professor of the Teaching of Mathematics
THE UNIVERSITY OF CHICAGO

Price 50 Cents



SUPPLEMENT FOR MODEL 803

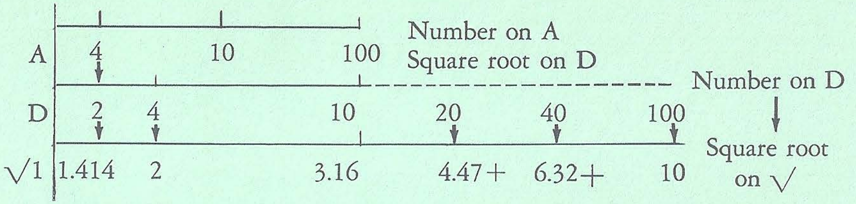
The Pickett Model 803 slide rule has four scales not discussed in the manual for Model 800, Model 500 and similar slide rules. These four scales are the following:

1. A "double length," or 20 inch scale for squares and square roots. This scale is labelled $\sqrt{}$, and is placed on the top of the stator on the "trig" side of the slide rule. The "right-hand portion" of this scale is placed under and "back-to-back" with the left-hand portion.
2. A special D scale which is "folded" at 0.43429 (that is, at $M = \log_{10} e$). This scale is labelled DF_M and is placed on the bottom of the stator on the "trig" side of the rule.
3. An extra 10 inch length of Log Log scale for which the range is 1.001 to 1.01. This scale is labelled LL0+, and it is placed on the top of Log Log side of the slide rule.
4. An extra 10 inch length of Log Log scale, for which the range is 0.999 to 0.99. This scale is labelled LL0—, and it is placed under and "back-to-back" with the LL0+ scale.

These scales provide an increase in the range, accuracy and convenience of the Model 803 in comparison with other slide rules which do not have these scales. The sections that follow tell why this is so and explain how to use these scales.

THE $\sqrt{}$ SCALE; SQUARE ROOTS AND SQUARES

The relation between the D scale and the square root (or $\sqrt{}$ scale) is the same as the relation between the A scale and the D scale. For example, if the hairline is set on a number (say 4) on the A scale, the square root of 4 (or 2) is under the hairline on the D scale. Similarly, if the hairline is set on a number (say 4) on the D scale, the square root of 4 is under the hairline on the $\sqrt{}$ scale. (See figure).



Slide rules have had A and B scales for hundreds of years. On a "10 inch" slide rule the A scale is about 5 inches long. It ends in the middle of the rule. There is room to put another section, just like the first, on the right. In finding square roots, if the left index of the A scale is read as 1, the middle index is read as 10 and the right index is read as 100. The D scale is twice as long as the A scale. On a "10 inch" slide rule, it is about 10 inches long.

Now observe that the $\sqrt{}$scale is twice as long as the D scale. On a "10 inch" slide rule, it is about 20 inches long. The right hand half of the $\sqrt{}$scale is placed *under* the left hand half. This is not a "new" scale — it is also old historically.

To find square roots or squares, when a slide rule has a $\sqrt{}$scale, the D scale can be used *instead of the A scale,* and then the $\sqrt{}$scale is used *instead of the D scale.* Because these scales are longer than the A scale, greater accuracy is possible.

Rule: **The square root of any number located on the D scale is found directly opposite it on the $\sqrt{}$ scale.**

Reading the Scales. The square root scale on the top of the stator is an enlargement of the D scale itself. The D scale has been "stretched" to double its former length. Because of this the square root scale seems to be cut off or to end with the square root of 10, which is about 3.16. To find the square root of numbers greater than 10 the lower $\sqrt{}$scale is used. This is really the rest of the stretched D scale. The small figure 2 near the left end is placed beside the mark for 3.2, and the number 4 is found nearly two inches farther to the right. In fact, if 16 is located on the D scale, the square root of 16, or 4, is directly above it on the lower $\sqrt{}$scale.

In general, the square root of a number between 1 and 10 is found on the upper square root scale. The square root of a number between 10 and 100 is found on the lower square root scale. If the number has an odd number of digits or zeros (1, 3, 5, 7,. . .), the upper $\sqrt{}$ scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8,. . .), the lower $\sqrt{}$ scale is used. The first three (or in some cases even four) figures of a number may be set on the D scale, and the first three (or four) figures of the square root are read directly from the proper square root scale.

The table below shows the number of digits or zeros in the number *N* and its square root.

	ZEROS				or	DIGITS				
	U L	U L	U L	U	L	U L	U L	U L	U L	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

The above table is reproduced on some models of Pickett Slide Rules.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

EXAMPLES:

(a) Find $\sqrt{248}$. Set the hairline on 248 of the D scale. This number has 3 (an *odd* number) digits. Therefore the figures in the square root are read from the upper $\sqrt{}$ scale as 1575. The result has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. Set the hairline on 563 of the D scale. The number has 6 (an *even* number) digits. Read the figures of the square root on the lower scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. Set the hairline on 1362 of the D scale. The number of *zeros* is 4 (an *even* number). Read the figures 369 on the lower scale. The result has 2 zeros, and is .00369.

Squaring **is the opposite of finding the square root. Locate the number on the proper $\sqrt{}$ scale and with the aid of the hairline read the square on the D scale.**

EXAMPLES:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the $\sqrt{}$ scale. On the D scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the $\sqrt{}$ scale. Find 394 above it on the D scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 628 was located on the lower of the $\sqrt{}$ scales, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the D scale read 645 above the 254 of the $\sqrt{}$ scale. The number has 3 zeros. Since 254 was located on the upper of the $\sqrt{}$ scales, the square has the odd number of digits, or 7. The result is 0.0000000645.

PROBLEMS	ANSWERS
1. $\sqrt{7.3}$	2.7
2. $\sqrt{73}$	8.54
3. $\sqrt{841}$	29
4. $\sqrt{0.062}$	0.249
5. $\sqrt{0.00000094}$	0.00097
6. $(3.95)^2$	15.6
7. $(48.2)^2$	2320
8. $(0.087)^2$	0.00757
9. $(0.00284)^2$	0.00000807
10 $(635000)^2$	4.03×10^{11}

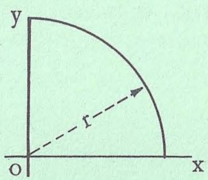
Since Model 803 has *both* an A scale and a $\sqrt{}$ scale, it is possible to read fourth roots and fourth powers directly.

Rule: If a number is located on the A scale its fourth root is opposite it on $\sqrt{}$. Conversely, if a number is located on $\sqrt{}$, its fourth power is opposite it on the A scale.

Example 1. Find $\sqrt[4]{13.6}$. Set hairline over 13.6 on A. Read answer 1.92 on $\sqrt{}$.

Example 2. Find 2.43^4. Set hairline over 2.43 on $\sqrt{}$. Read answer 34.9 on A.

Example 3. Find the moment of inertia of the quarter circle (shown in the figure) about the line OX when the radius r is 3.5 in. The formula in this case is $I_x = \frac{\pi r^4}{16}$.



Set the hairline of the indicator over 3.5 on $\sqrt{}$. Move the slide so 16 on B is under the hairline. Move the indicator over π on B. Read the result $I_x = 29.5$ in.^{4} on the A scale.

Example 4. The quarter circle shown in the figure for Example 3 is to have a moment of inertia about the line OX of 24.3 in.^{4}. What length must the radius be? Solving the formula given in Example 3 for r, we obtain $r = \sqrt[4]{\frac{16\, I_x}{\pi}}$.

Hence $r = \sqrt[4]{\frac{16 \times 24.3}{\pi}}$ Set π on B under 16 on the A scale. Move the indicator over 24.3 on the B scale. Read $r = 1.875$ in. on the $\sqrt{}$ scale under the hairline.

In computing with Model 803, the $\sqrt{}$ scale not only permits greater accuracy, but also greater convenience.

Example: Suppose you need to draw a graph of $\rho^2 = 8 \cos \theta$ in polar coordinates. It is necessary to compute $\rho = \pm\sqrt{8 \cos. \theta}$. Set the right index of the C scale over 8 of the D scale. Move hairline over θ on the S scale, reading from right to left. Read ρ directly on the $\sqrt{}$ scale. Thus a table like the one at the left can be quickly and accurately computed with *only one setting of the slide.*

θ	ρ
0	±2.828
10	±2.807
20	±2.742
30	±2.632
40	±2.475
50	±2.267
60	±2.000

The example above is much more difficult to do using the A scale on a modern slide rule in which the S scale is based on the D scale. Also, less accuracy is possible. With a rule of older design in which the S scale is based on the A scale, the work is done by the same method (using the A, S, and D scales) but with less accuracy.

THE DF_M SCALE

The Log Log scales on Model 803 are arranged so that logarithms to base e can be read from the D scale. Thus, if the indicator is set over 15 on LL3+, then $\log_e 15 = 2.708$ can be read on the D scale.

Often it is convenient or necessary to find the logarithm to base 10. This can, of course, be done by using the L scale. Thus if 15 is set on C, the mantissa of $\log_{10} 15$, or 0.176 is on L. The characteristic of $\log_{10} 15$, determined by rule, is 1, so $\log_{10} 15 = 1.176$. It is more convenient to read the entire logarithm (both characteristic and mantissa) directly, and this can be done on the DF_M scale. With the hairline set on 15 of LL3+, $\log_{10} 15$ can be read *directly* on DF_M as 1.176. The DF_M is a standard D scale which is folded at 0.43429, which is $\log_{10} e$. It automatically converts logarithms to base e, when located on D, to logarithms base 10 on DF_M, and conversely.

EXAMPLES:

(a) Find $\log_{10} 4.7$. Set hairline over 4.7 of LL3+. Read 0.672 on DF_M.

(b) Find $\log_{10} 0.213$. Let hairline over 0.213 of LL3–. (Note 0.213 is the reciprocal of 4.7, so the setting is the same as for example (a), above). Read 0.672 on DF_M. Since 0.213 is less than 1, the logarithm is negative. Then $\log_{10} 0.213 = -.672$ or $9.328-10$.

(c) Find $\log_{10} 1.36^{2.4}$. Set indicator hairline over 1.36 on LL2+. Set the left index of C under the hairline. Move hairline over 2.4 of C. Read answer on DF_M as 0.3205.

(d) Find X if $\log_{10} X = 1.673$. Set hairline over 1.673 on DF_M. Read 47.1 on LL3+.

PROBLEMS	ANSWERS
(a) $\log_{10} 4$	0.602
(b) $\log_{10} 1.15$	0.0607
(c) $\log_{10} 1.02$	0.00860
(d) $\log_{10} 30$	1.477
(e) $\log_{10} 1.405$	0.1477
(f) $\log_{10} 1.0346$	0.01477
(g) $\log_{10} 0.05$	-1.301 or $8.699-10$
(h) $\log_{10} 0.63$	-0.201 or $9.799-10$
(i) $\log_{10} 0.946$	-0.0241 or $9.9759-10$