

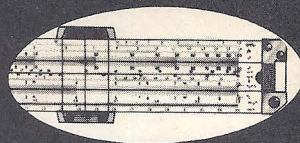
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electronics

CLEVELAND INSTITUTE OF ELECTRONICS / CLEVELAND, OHIO

Electronics and
Your Slide Rule
Part IV

3125-1



An AUTO-PROGRAMMED™ Lesson

ABOUT THE AUTHOR

Through over 15 years experience in helping students learn through home study, Mr. Geiger has obtained an intimate understanding of the problems facing home-study students. He has used this knowledge to make many improvements in our teaching methods. Mr. Geiger believes that students learn fastest when they actively participate in the lesson, rather than just reading it.

Mr. Geiger edits much of our new lesson material, polishing up the manuscripts we receive from subject-matter experts so that they are easily readable, contain only training useful to the student in practical work, and are written so as to teach, rather than merely presenting information.

Mr. Geiger's book, *Successful Preparation for FCC License Examinations* (published by Prentice-Hall), was chosen by the American Institute of Graphic Arts as one of the outstanding text books of the year.



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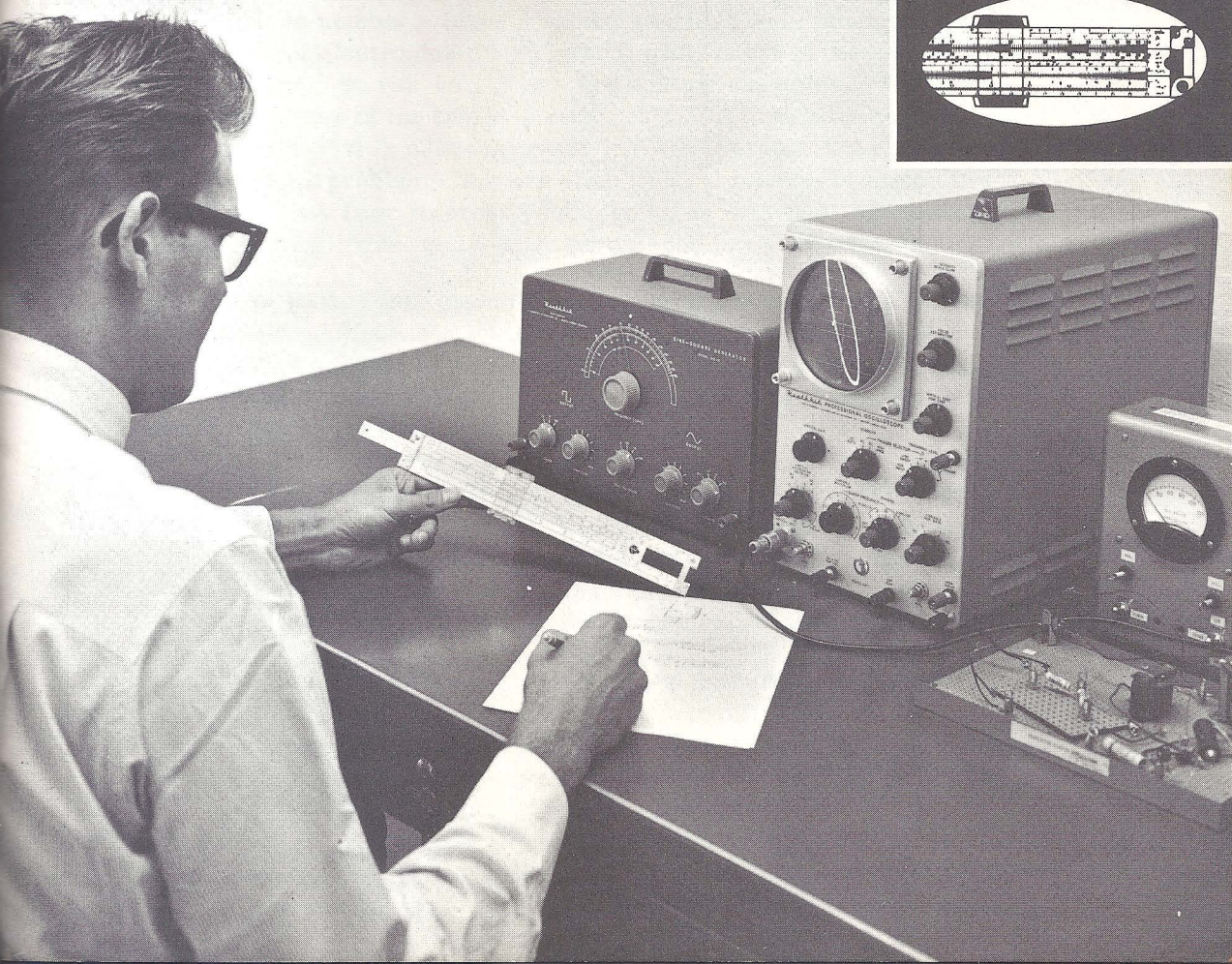
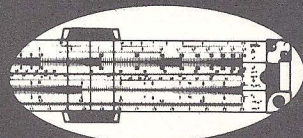
CLEVELAND INSTITUTE OF ELECTRONICS

Electronics and Your Slide Rule

Solving Trigonometry Problems with Electronic Applications

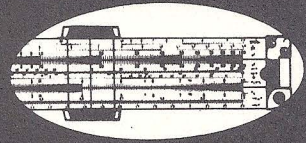
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3125-1



In this lesson you will learn...

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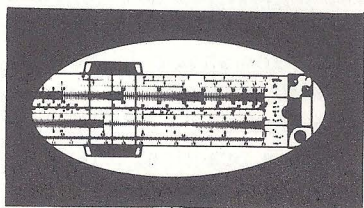
A chat with your instructor

This lesson completes your slide rule training. If you have worked all the practice problems up to and including this lesson, you are well on your way to slide rule proficiency. You should by now be able to work problems on the slide rule without feeling a certain distrust of your answers. You have broken free of the need of pencil calculations... now when you are faced with a practical problem, you can reach for your slide rule for quick answers.

The applications of the principle of the right triangle in electronics are great in number. Since most such applications require a good knowledge of electronics theory, which many users of these lessons will not have, we can only give a few simple examples. The most widespread use of the trigonometric scales in electronics is in connection with what is called operator j. With operator j the solution to a-c circuit problems is much simplified. I mention operator j in the hopes that I can interest you in going on with your electronics training--and consequently on to better jobs in electronics.

The latter part of this lesson is an appendix that covers the use of the $\ln$ scale. Since this scale is of most value to those with advanced electronics training, it is optional as to whether or not you study this section. A knowledge of the $\ln$ scale is not needed in order to answer the examination on this lesson.

We are always interested in hearing from graduates of our slide rule training program. Particularly welcome are comments on how we might improve your Electronics Slide Rule and our lesson material. And don't hesitate to let me know if I can be of help in answering any questions you may still have on the proper use of your slide rule.



Electronics and Your Slide Rule Part IV

SOLUTIONS OF THE RIGHT TRIANGLE

The general right triangle having hypotenuse c , legs a and b , and angles A , B , and $C = 90^\circ$ is again pictured in Fig. 31. When any one side and one acute angle are known, or when any two sides are known, the remaining parts of the triangle can be found with your slide rule. This process of finding all the sides and angles of the triangle is called "solving" the triangle.

- 37** SOLVING THE RIGHT TRIANGLE WHEN THE HYPOTENUSE c AND ONE ACUTE ANGLE, A OR B , ARE KNOWN... The operations required in solving the right triangle when one side and one angle are known are just the combined operations of multiplication

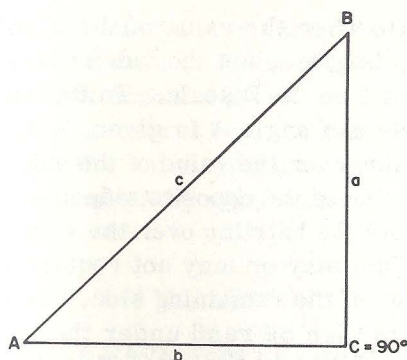


Fig. 31. The general right triangle.

and division with trigonometric functions, operations with which you are already familiar. For example, when the hypotenuse c and the angle A are known, to find the legs of the triangle, a and b , we use the formulas $b = c \cos A$ and $a = c \sin A$. Thus in Fig. 31, if $c = 28.0$ and $A = 27^\circ$, we have $b = 28.0 \cos 27^\circ$ and $a = 28.0 \sin 27^\circ$. The combined multiplication operation, $28.0 \cos 27^\circ$, is performed as usual, by setting the right index of the slide over 28.0 on the D scale and the hairline over 27 on the red numbered S scale, then reading $b = 28.0 \cos 27^\circ = 24.95$ on the D scale. The slide is in the correct position to perform the operation $28.0 \sin 27^\circ$ and only the hairline has to be moved. Without moving the slide, perform this operation by moving the hairline over 27 on the black numbered S scale and read the value of a on the D scale under the hairline; $a = 28.0 \sin 27^\circ = 12.7$. The angle B can be found by subtracting 27° from 90° ; $B = 90^\circ - 27^\circ = 63^\circ$.

In some cases it is not possible to perform the second operation without moving the slide. For instance, if in the preceding example A was 20° instead of 27° , the slide would not be in the correct position to perform the operation $b = 28.0 \sin 20^\circ$, since 20° on the black portion of the S scale would be out of the range of the D scale. In these cases the slide must be moved so that the other index of the slide covers the value of the hypotenuse on the D scale. Hence, to find $b = 28.0 \sin 20^\circ$, you must move the slide so that the left index rests over 28.0 on the D scale.

To solve the triangle when the value of the hypotenuse and one acute angle A or B , is given, set the index of the slide over the value of the hypotenuse on the D scale. To find the side opposite the given angle (side a if angle A is given, side b if angle B is given), set the hairline over the value of the angle on the black S scale and read the value of the opposite side under the hairline of the D scale. Then set the hairline over the value of the angle on the red S scale. (This may or may not require the resetting of the slide.) The value of the remaining side, which is adjacent to the given angle, can then be read under the hairline on the D scale. The decimal is placed by noting that the two legs of the triangle are never greater than the hypotenuse and never less than one tenth the hypotenuse for the angles within the range of the S and T scales. The remaining angle of the triangle can be found by subtracting the given angle from 90° .

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Example...47

In Fig. 32 the right triangle is shown for $c = 20$, $B = 25^\circ$. Solve this triangle.

Solution...

- 1...Set the left index of the slide opposite 20 on the D scale.
- 2...Here side b is the side opposite the known angle $B = 25^\circ$. To find b set the hairline over 25° on the black S scale and read 8.46 on the D scale. Since the value of b must lie between 20 and 2.0, $b = 8.46$.

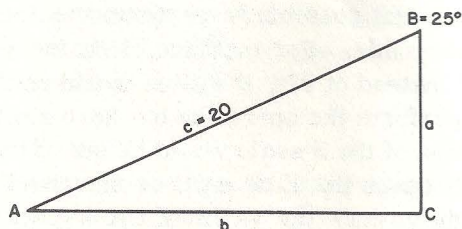


Fig. 32. The triangle to be used in Example 47.

- 3... To find the value of the adjacent side a the slide must be reset. Move the slide so that the right index is opposite 20 on the D scale and move the hairline over 25 on the red numbered S scale. Read the value of a on the D scale; $a = 18.13$.
- 4... Find angle A by subtracting 25° from 90° ; $A = 90^\circ - 25^\circ = 65^\circ$.
-

WHAT HAVE YOU LEARNED?

Solve the right triangle of Fig. 31 for the following cases:

- | | |
|------------------------------------|-------------------------------------|
| 1... $c = 14.6$, $A = 18^\circ$ | 7... $c = 249$, $A = 28.6^\circ$ |
| 2... $c = 17.2$, $B = 32.6^\circ$ | 8... $c = 68.1$, $A = 32.7^\circ$ |
| 3... $c = 39.7$, $B = 53.2^\circ$ | 9... $c = 43.5$, $B = 46.5^\circ$ |
| 4... $c = 108$, $A = 64.1^\circ$ | 10... $c = 50.3$, $B = 55^\circ$ |
| 5... $c = 8.61$, $B = 24.3^\circ$ | 11... $c = 72.1$, $B = 19.7^\circ$ |
| 6... $c = 1.75$, $A = 36.4^\circ$ | 12... $c = 432$, $A = 25.5^\circ$ |

ANSWERS

- | | |
|--|--|
| 1... $B = 90^\circ - 18^\circ = 72^\circ$
$a = 14.6 \sin 18^\circ = 4.51$
$b = 14.6 \cos 18^\circ = 13.89$ | 8... $B = 57.3^\circ$
$a = 36.8$
$b = 57.3$ |
| 2... $A = 57.4^\circ$
$a = 14.5$
$b = 9.27$ | 9... $A = 43.5^\circ$
$a = 29.9$
$b = 31.55$ |
| 3... $A = 36.8^\circ$
$a = 23.8$
$b = 31.8$ | 10... $A = 35^\circ$
$a = 28.85$
$b = 41.2$ |
| 4... $B = 25.9^\circ$
$a = 97.1$
$b = 47.2$ | 11... $A = 70.3^\circ$
$a = 67.9$
$b = 24.3$ |
| 5... $A = 65.7^\circ$
$a = 7.84$
$b = 3.54$ | 12... $B = 64.5^\circ$
$a = 186$
$b = 389$ |
| 6... $B = 53.6^\circ$
$a = 1.038$
$b = 1.41$ | |
| 7... $B = 61.4^\circ$
$a = 119$
$b = 218.6$ | |

38 SOLVING THE RIGHT TRIANGLE WHEN ONE LEG, a OR b , AND THE OPPOSITE ANGLE ARE KNOWN...In Fig. 31, when the side a and the angle A opposite this side are known, the hypotenuse c can be found by the formula, $c = a/\sin A$. Once c has been found, the other leg of the triangle, b , can be obtained by the formula, $b = c \cos A$.

For the case where $a = 25$ and $A = 35^\circ$, we have $c = 25/\sin 35^\circ$ and $b = c \cos 35^\circ$. First find c by setting the hairline over 25 on the D scale and moving the slide so that the hairline also covers 35 on the black numbered S scale. Read $c = 43.6$ on the D scale under the right index of the slide. Note that the slide is in the correct position to find $b = 43.6 \cos 35^\circ$ and need not be moved in this case. However, in some cases the slide must be reset to perform this second operation. Set the hairline over 35 on the red S scale and read $b = 35.7$ under the hairline on the D scale. Find angle B by subtraction; $B = 90^\circ - 35^\circ = 55^\circ$.

In general, when one acute angle and the side opposite this angle is known, set the hairline over the value of the side on the D scale. Move the slide so that the hairline covers the value of the angle on the black numbered S scale. Read the value of the hypotenuse on the D scale under the index of the slide. This value will be no less than the value of the known side and no greater than 10 times it. Then move the hairline so that it covers the value of the known angle on the red numbered S scale. (This may or may not require a resetting of the slide so that the other index covers the value of the hypotenuse on the D scale.) Read the value of the remaining side, which is adjacent to the known angle, under the hairline on the D scale. The unknown angle is found by subtracting the known angle from 90° .

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Example...48

In Fig. 33 the right triangle is shown for $b = 55$ and $B = 32^\circ$. Solve this triangle.

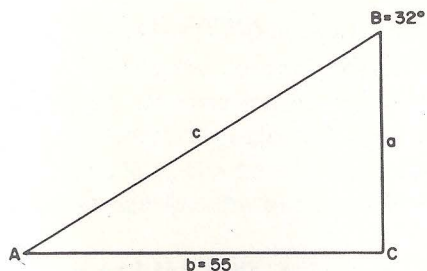


Fig. 33. The triangle to be used for Example 48.

Solution...

- 1...Set the hairline over 55 on the D scale.
- 2...Move the slide so that the hairline is over 32 on the black S scale and read 1038 on the D scale under the index of the slide. Since the value of c must lie between 55 and 550, $c = 103.8$.
- 3...Move the hairline so that it covers 32 on the red S scale, this can be done without resetting the slide in this case. Read the value of the remaining side, a , below the hairline on the D scale. The value of a must lie between 10.4 and 104, hence $a = 88.0$.
- 4...Find angle A by subtraction; $A = 90^\circ - 32^\circ = 58^\circ$.

.....

WHAT HAVE YOU LEARNED?

Solve the right triangle of Fig. 31 in the following cases:

- | | |
|------------------------------------|-------------------------------------|
| 1... $a = 14.2$, $A = 24^\circ$ | 7... $b = 42.3$, $B = 28^\circ$ |
| 2... $b = 138$, $B = 39.5^\circ$ | 8... $b = 28.5$, $B = 63.1^\circ$ |
| 3... $a = 65.1$, $A = 44.2^\circ$ | 9... $a = 278$, $A = 75^\circ$ |
| 4... $a = 1.39$, $A = 22.4^\circ$ | 10... $a = 72.5$, $A = 16.1^\circ$ |
| 5... $b = 34.5$, $B = 13^\circ$ | 11... $b = 11.1$, $B = 35.2^\circ$ |
| 6... $b = 654$, $B = 44.1^\circ$ | 12... $a = 709$, $A = 21.3^\circ$ |

- | | |
|-----------------------|------------------------|
| 1... $B = 66^\circ$ | 7... $A = 62^\circ$ |
| $c = 34.9$ | $c = 90.1$ |
| $d = 31.9$ | $a = 79.5$ |
| 2... $A = 50.5^\circ$ | 8... $A = 26.9^\circ$ |
| $c = 217$ | $c = 31.95$ |
| $a = 167.4$ | $a = 14.45$ |
| 3... $B = 45.8^\circ$ | 9... $B = 15^\circ$ |
| $c = 93.5$ | $c = 288$ |
| $b = 67.0$ | $b = 74.5$ |
| 4... $B = 67.6^\circ$ | 10... $B = 73.9^\circ$ |
| $c = 3.65$ | $c = 261.5$ |
| $b = 3.37$ | $b = 251$ |
| 5... $A = 77^\circ$ | 11... $A = 54.8^\circ$ |
| $c = 153.4$ | $c = 19.26$ |
| $a = 149$ | $a = 15.73$ |
| 6... $A = 45.9^\circ$ | 12... $B = 68.7^\circ$ |
| $c = 939$ | $c = 1951$ |
| $a = 674$ | $b = 1818$ |

39 SOLVING THE RIGHT TRIANGLE WHEN ONE LEG, a OR b , AND THE ADJACENT ANGLE ARE KNOWN...With reference to Fig. 31, when side a and the angle B adjacent to this side are known, the hypotenuse c can be found by the formula $c = a/\cos B$. Once c has been found, the other leg of the triangle can be found by the formula $b = c \sin B$. Note that these formulas are similar to those of the previous section where side a and angle A were given. For this reason the operations are quite similar. In fact, you will find the operations are the same except the roles of the red and black numbered S scales are interchanged.

To see this similarity, consider the example given in the beginning of the previous section. Here $a = 25$ and $B = 55^\circ$. Find c by setting the hairline over 25 on the D scale and this time move the slide so that the hairline also covers 55 on the red S scale. The value $c = 43.6$ is read under the right index of the slide on the D scale. In the previous section this operation was performed by setting the slide such that the hairline covered the value of $A = 35^\circ$

on the black S scale----you can see this gives the same slide rule settings. The slide is now in the correct position to perform the operation $b = 43.6 \sin 55^\circ$ and need not be reset. Set the hairline over 55 on the black numbered S scale and read the value of b on the D scale under the hairline; $b = 35.7$. Here again the settings are identical to the settings for the operation $b = 43.6 \cos 35^\circ$.

When one acute angle and the side adjacent to this angle are known, set the hairline over the value of the known side on the D scale. Move the slide so that the hairline covers the value of the angle on the red S scale. Read the value of the hypotenuse on the D scale under the index of the slide. Then move the hairline so that it covers the value of the known angle on the red S scale. (This may or may not require a resetting of the slide.) Read the value of the remaining side under the hairline on the D scale. Find the unknown angle by subtracting the known angle from 90° .

.....

Example...49

In Fig. 34 the right triangle is shown for $b = 30$ and $A = 60^\circ$. Solve this triangle.

Solution...

- 1...Set the hairline over 30 on the D scale.
- 2...Move the slide so that the hairline covers 60 on the red S scale and read 600 under the index of the slide on the D scale. The value of c must lie between 30 and 300, hence $c = 60.0$.
- 3...Move the hairline so that it covers 60 on the black S scale. This can be done without resetting the slide. Read the value of a on the D scale. This value must lie between 6.00 and 60.0, hence $a = 52.0$.
- 4...Find angle B by subtraction; $B = 90^\circ - 60^\circ = 30^\circ$.

WHAT HAVE YOU LEARNED?

Solve the right triangle of Fig. 31 in the following cases:

- | | |
|------------------------------------|------------------------------------|
| 1... $a = 36.7$, $B = 52.3^\circ$ | 3... $b = 108$, $A = 44.1^\circ$ |
| 2... $a = 22.2$, $B = 19.5^\circ$ | 4... $a = 39.1$, $B = 14.6^\circ$ |

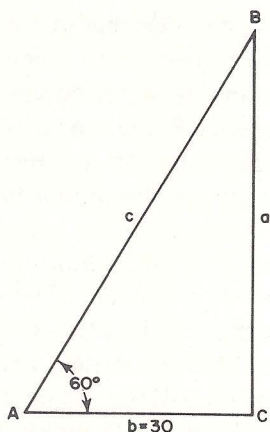


Fig. 34. The right triangle with one acute angle and an adjacent side given.

5... $b = 1.52$, $A = 24.7^\circ$

9... $b = 21.5$, $A = 62.3^\circ$

6... $b = 50.2$, $A = 65.1^\circ$

10... $a = 2.87$, $B = 41.2^\circ$

7... $b = 1.73$, $A = 25^\circ$

11... $b = 109$, $A = 12.4^\circ$

8... $a = 15.7$, $B = 39.1^\circ$

12... $a = 67.5$, $B = 24.3^\circ$

ANSWERS

1... $A = 37.7^\circ$

$c = 60$

$b = 47.5$

2... $A = 70.5^\circ$

$c = 23.55$

$b = 7.86$

3... $B = 45.9^\circ$

$c = 150.4$

$a = 104.7$

4... $A = 75.4^\circ$

$c = 40.4$

$b = 10.19$

5... $B = 65.3^\circ$

$c = 1.673$

$a = 0.699$

6... $B = 24.9^\circ$

$c = 119.2$

$a = 108$

7... $B = 65^\circ$

$c = 1.909$

$a = .807$

8... $A = 50.9^\circ$

$c = 20.23$

$b = 12.75$

9... $B = 27.7^\circ$

$c = 46.25$

$a = 40.95$

10... $A = 48.8^\circ$

$c = 3.185$

$b = 2.51$

11... $B = 77.6^\circ$

$c = 111.6$

$a = 23.98$

12... $A = 65.7^\circ$

$c = 74.2$

$b = 30.5$

SOLVING THE RIGHT TRIANGLE WHEN THE HYPOTENUSE c AND ONE OTHER SIDE, a OR b , ARE GIVEN... When c and, say, b are known, the triangle can be solved in the following way. Since $b/c = \cos A$, $\cos A$ is known, hence A can be found. Angle B is obtained by subtracting A from 90° and a is obtained from the formula $a = c \sin A$. These operations will now be combined to give the most efficient way of solving the right triangle when the hypotenuse and one other side are known.

Consider a triangle in which $c = 5$ and $b = 4$. Set the right index of the slide opposite 5 on the D scale and the hairline over 4 on the D scale. Notice that the hairline now covers the value $4/5 = 0.80$ on the C scale. Here you have actually divided 4 by 5 using the C and D scales in a new way. Since the index of the slide rests over 5 and the hairline covers 4 on the D scale, the number under the hairline on the C scale is the number which multiplies 5 to give 4, or just $4/5$. This type of division has the advantage of yielding the answer on the C scale instead of the D scale. The ratio $4/5 = 0.80$, under the hairline is $\cos A = b/c$, hence A can be read directly as 36.9° under the hairline on the red numbered portion of the S scale. Since the black numbers of the S scale are 90° minus the red numbers, the angle $B = 90^\circ - A$ can be read under the hairline on the black numbered portion of the S scale; $B = 53.1^\circ$. Do not move the slide since it is in the correct position to perform the operation $a = c \sin A = 5 \sin 36.9^\circ$. Move the hairline over 36.9° on the black numbered S scale and read $a = 3.00$ on the D scale under the hairline. In placing the decimal point we have again made use of the fact that the legs of the right triangle are never less than one tenth the hypotenuse and never greater than it for angles within the range of the S scale.

To solve the right triangle when the hypotenuse and one other side are given, set the index of the slide over the value of the hypotenuse on the D scale and then set the hairline over the value of the known side on the D scale. Read the value of the angle between the hypotenuse and this known side on the red S scale. Read the value of the other acute angle on the black S scale. Now set the hairline over the value of the first angle you found on the black

S scale. (This may require a resetting of the slide.) Read the value of the unknown side under the hairline on the D scale.

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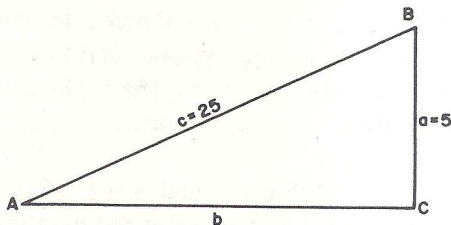


Fig. 35. The case where the hypotenuse and one side are known.

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Example... 50

In Fig. 35 the right triangle is shown for $c = 25$ and $a = 5$. Solve this triangle.

Solution...

- 1...Set the index over 25 on the D scale.
 - 2...Here the angle B is the angle between the hypotenuse and the known side a. Read the value of B, 78.48° , on the S scale.
 - 3...Read angle A = 11.52° on the black S scale.
 - 4...Set the hairline over 78.48° on the black S scale. Read the value of the unknown side, $b = 24.5$, on the D scale.
-

WHAT HAVE YOU LEARNED?

Solve the right triangle of Fig. 31 in the following cases:

- | | |
|------------------------------|------------------------------|
| 1... $c = 35.0$, $b = 15.0$ | 5... $c = 16.3$, $a = 9.12$ |
| 2... $c = 128$, $a = 54.3$ | 6... $c = 44.5$, $b = 20.2$ |
| 3... $c = 11.1$, $a = 7.04$ | 7... $c = 112$, $b = 39.0$ |
| 4... $c = 115$, $b = 24.6$ | 8... $c = 32.1$, $a = 28.6$ |

$$9...c = 42.6, b = 11.4$$

$$11...c = 1.04, b = .213$$

95

$$10...c = 75.5, a = 38.9$$

$$12...c = 8.64, a = 4.72$$

ANSWERS

$$1...A = 64.6^\circ$$

$$B = 25.4^\circ$$

$$a = 31.6$$

$$2...A = 25.1^\circ$$

$$B = 64.9^\circ$$

$$b = 115.9$$

$$3...A = 39.4^\circ$$

$$B = 50.6^\circ$$

$$b = 8.58$$

$$4...A = 77.65^\circ$$

$$B = 12.35^\circ$$

$$a = 112.2$$

$$5...A = 34.0^\circ$$

$$B = 56.0^\circ$$

$$b = 13.5$$

$$6...A = 63.0^\circ$$

$$B = 27.0^\circ$$

$$a = 39.65$$

$$7...A = 69.6^\circ$$

$$B = 20.4^\circ$$

$$a = 105$$

$$8...A = 63.1^\circ$$

$$B = 26.9^\circ$$

$$b = 14.5$$

$$9...A = 74.47^\circ$$

$$B = 15.53^\circ$$

$$a = 41$$

$$10...A = 31.0^\circ$$

$$B = 59.0^\circ$$

$$b = 64.7$$

$$11...A = 78.19^\circ$$

$$B = 11.71^\circ$$

$$a = 1.019$$

$$12...A = 33.1^\circ$$

$$B = 56.9^\circ$$

$$b = 7.24$$

41

SOLVING THE RIGHT TRIANGLE WHEN THE TWO LEGS, a AND b , ARE GIVEN... If a and b are known, $\tan A = a/b$ is known and A can be found. Then to find the hypotenuse c we make use of the formula $c = a/\sin A$, and to find B we subtract A from 90° . For example, consider the triangle of Fig. 31 with $a = 3$ and $b = 6$. Set the right index of the slide over 6 on the D scale and then the hairline over 3 on the D scale. The ratio, $a/b = \tan A = 0.50$, then appears on the C scale. Since this ratio is less than one, A must be less than 45° and can be read directly under the hairline on the black numbered T scale as $A = 26.56^\circ$. Since the red numbers of the T scale are 90° minus the black one, angle B can be read on the red numbered portion as 63.44° . The hairline is in the correct position to perform the operation $c = 3/\sin 26.56^\circ$ and

only the slide has to be moved. Move the slide so that the hairline covers 26.56° on the black S scale. The value of the hypotenuse, 6.71, then appears opposite the index of the slide on the D scale.

In general, to solve the right triangle when the two legs are given set the index of the slide over the value of the larger of the two sides on the D scale. Next, set the hairline over the value of the smaller of the two sides, also on the D scale. Read the value of the angle between the larger side and the hypotenuse on the black T scale. Read the value of the other acute angle on the red T scale. Without moving the hairline, move the slide so that the hairline covers the value of the first angle on the black S scale or the second angle on the red S scale. Read the value of the hypotenuse under the index of the slide on the D scale.

.....

Example... 51

In Fig. 31 assume that $b = 8$ and $a = 39$. Solve the triangle.

Solution...

- 1...Set the left index of the slide over 39 on the D scale.
 - 2...Set the hairline over 8 on the D scale and read the value of B on the black T scale; $B = 11.59^\circ$. Read the value of A on the red T scale; $A = 78.41^\circ$.
 - 3...Without moving the hairline, move the slide so that the hairline covers 11.59 on the black S scale. Read the value of c under the left index of the slide; $c = 39.8$.
-

WHAT HAVE YOU LEARNED?

Solve the right triangle of Fig. 31 in the following cases:

- | | |
|------------------------------|------------------------------|
| 1... $a = 27.2$, $b = 18.9$ | 3... $a = 7.23$, $b = 25.3$ |
| 2... $a = 108$, $b = 57.5$ | 4... $a = 1.73$, $b = 1.05$ |

$$5...a = 31.4, b = 14.2$$

$$9...a = 33.6, b = 98.9$$

97

$$6...a = 128, b = 39.1$$

$$10...a = 16.2, b = 9.12$$

$$7...a = .111, b = .789$$

$$11...a = 32.1, b = 49.7$$

$$8...a = 75.6, b = 106$$

$$12...a = 141, b = 200$$

ANSWERS

$$1...A = 55.2^\circ$$

$$B = 34.8^\circ$$

$$c = 33.1$$

$$2...A = 62.0^\circ$$

$$B = 28.0^\circ$$

$$c = 122.5$$

$$3...A = 15.92^\circ$$

$$B = 74.08^\circ$$

$$c = 26.3$$

$$4...A = 58.75^\circ$$

$$B = 31.25^\circ$$

$$c = 2.025$$

$$5...A = 65.68^\circ$$

$$B = 24.32^\circ$$

$$c = 34.5$$

$$6...A = 73.01^\circ$$

$$B = 16.99^\circ$$

$$c = 133.8$$

$$7...A = 8.01^\circ$$

$$B = 81.99^\circ$$

$$c = .797$$

$$8...A = 35.5^\circ$$

$$B = 54.5^\circ$$

$$c = 130.2$$

$$9...A = 18.76^\circ$$

$$B = 71.24^\circ$$

$$c = 104.5$$

$$10...A = 60.64^\circ$$

$$B = 29.36^\circ$$

$$c = 18.6$$

$$11...A = 32.9^\circ$$

$$B = 57.1^\circ$$

$$c = 59.2$$

$$12...A = 35.19^\circ$$

$$B = 54.81^\circ$$

$$c = 245$$

42

FINDING TRIGONOMETRIC FUNCTIONS OF SMALL ANGLES...

The sine and tangent of small angles (below the range of scales S and T) are approximately equal and can be found approximately by multiplying the sine of the angles in degrees by the factor 0.01745. This is the method used for finding the functions of angles too small to be found on the S or T scales. It is not necessary to remember the constant value 0.01745 because it is permanently marked on the C, D, and CI scales of your slide rule and identified by a small degree mark ($^\circ$).

To find the sine of 0.7° , place the right index of scale C opposite 7 on scale D, and then opposite the degree mark between 1.7 and

98. 1.8 on scale C, read 122 on scale D. Thus the sine of 0.7° is 0.0122. This is also the tangent of 0.7° .

The sine and tangent values for angles below the range of the S and T scales have a 0 between the decimal point and the first significant figure. If the angle size was less than 0.573° there would be more than one zero between the decimal point and the first figure, but it is unlikely that you will be working with angles that small.

As another example with small angles, suppose we wish to find the angle whose tangent is 0.0838. Since there is a zero between the decimal point and the first figure of the value, we know that the angle required is below the range of the S or T scale. To find the tangent of a small angle we multiply the angle size by 0.01745. Conversely, to find the angle corresponding to a given tangent value, we would divide that value by 0.01745. Thus, we set the degree mark on scale C opposite 838 on scale D and then read 4.8° , the answer, opposite the C index on scale D.

WHAT HAVE YOU LEARNED?

1... Find the sine and tangent of each of the following angles:

- | | |
|------------------|------------------|
| (a) 1.72° | (e) 3.82° |
| (b) 3.59° | (f) 2.21° |
| (c) 4.86° | (g) 1.68° |
| (d) 1.53° | (h) 5.31° |

2... Find the angle A in each of the following problems:

- | | |
|----------------------|----------------------|
| (a) $\tan A = .0437$ | (e) $\sin A = .0296$ |
| (b) $\sin A = .0252$ | (f) $\tan A = .0341$ |
| (c) $\tan A = .0621$ | (g) $\sin A = .0557$ |
| (d) $\tan A = .0843$ | (h) $\tan A = .0201$ |

- | | |
|-----------------|------------|
| 1... (a) .0300 | (e) .0666 |
| (b) .0626 | (f) .03856 |
| (c) .0848 | (g) .0293 |
| (d) .0267 | (h) .0926 |
| 2... (a) 2.505° | (e) 1.695° |
| (b) 1.445° | (f) 1.955° |
| (c) 3.56° | (g) 3.19° |
| (d) 4.83° | (h) 1.152° |

43

SOLVING CIRCUIT PROBLEMS BY TRIANGLE RULES... The trigonometric scales of the slide rule are extremely useful in working a-c circuit problems. As an example, consider the series RLC circuit. From a-c circuit theory it is known that a phasor impedance triangle can be drawn for such a circuit. This triangle and its circuit are shown in Fig. 36. The resistance R and the total reactance $X = X_L - X_C$ make up the two legs of the triangle. The impedance of the circuit, Z , is the hypotenuse, and the angle Θ between R and Z represents the angle by which the current I leads the voltage E . The solutions to the right triangle discussed in the previous sections can be directly applied to this impedance triangle.

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Example...52

In the circuit of Fig. 36 let $R = 50 \Omega$, $X_L = 80 \Omega$, $X_C = 65 \Omega$, and $I = .51$ amp. Find the impedance Z , the phase angle Θ and the voltage E .

Solution...

Here the two legs of the impedance triangle are known; $R = 50 \Omega$ and $X = 80 - 65 = 15 \Omega$, the hypotenuse Z and one acute angle, θ , must be found. With reference to the previous sections

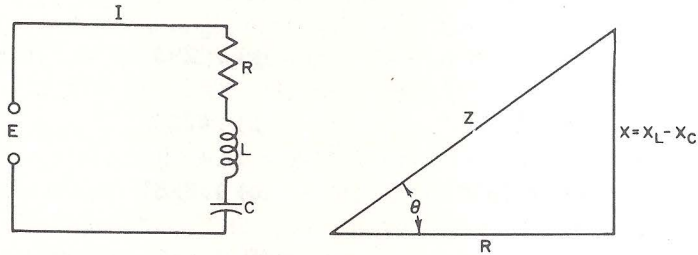


Fig. 36. The series circuit and its impedance triangle.

the triangle is solved as follows:

- (a) Set the right index of the slide over the value $R = 50$ on the D scale and the hairline over $X = 15$ also on the D scale.
- (b) The angle θ is the angle between R , (which is the larger of the two legs) and Z . Hence this angle is read on the black T scale as $\theta = 16.7^\circ$.
- (c) Without moving the hairline, set the slide so that the hairline covers 16.7 on the black S scale and read the value of Z on the D scale; $Z = 52.3 \Omega$.
- (d) Do not move the slide since it is in the correct position to find the voltage: $E = IZ$. Set the hairline over $I = .51$ on the C scale and read $E = 26.6$ volts (52.3×0.51). on the D scale.

In the circuit of Fig. 36, the power which the circuit dissipates is given by the formula $P = EI \cos \theta$. The power factor of the circuit is $\cos \theta = \frac{P}{EI}$ and represents the ratio of the actual power supplied to the circuit and the product of the impressed voltage and current. The essential a-c circuit formulas involving circuit power are permanently marked on the back of your slide rule under the heading "Ohm's Law Formulas, AC Circuits". By means of these formulas, problems involving circuit power can also be handled with your slide rule by methods with which you are already familiar.

.....

Example...53

The series circuit of Fig. 36 is connected to a supply of $E = 110$ volts. If $R = 1,100\ \Omega$ and $X = 1,500\ \Omega$, what is the power factor of this circuit? How much power will the resistance R dissipate?

Solution...

The power factor is $\cos \theta$ and can be found from X and R . The impedance Z can also be found from X and R , and then the formula

$P = \frac{E^2 \cos \theta}{Z}$ (which can be found on the back of the rule) can be used to find the power dissipated.

- (a) Set the right index of the slide over 1,500 on the D scale and the hairline over 1,100 on the D scale. Read $\theta = 53.7^\circ$ on the red T scale.
- (b) Set the slide so that the hairline covers 53.7 on the red S scale. Read the value of Z on the D scale as $Z = 1,860\ \Omega$.
- (c) Notice that since the hairline now lies over the value of θ on the red S scale, the value of $\cos \theta$ must lie under the hairline on the C scale; $\cos \theta = .592$. The power factor is then 59.2%.
- (d) The circuit power is the power which R dissipates and can now be calculated:

$$P = \frac{E^2 \cos \theta}{Z} = \frac{(110)^2 (.592)}{(1,860)} = 3.85 \text{ watts.}$$

In the preceeding examples the trigonometric scales were used with a minimum number of settings. In problems such as this, where a number of combined operations are involved, there is no general rule which will give the minimum number of settings. You should, however, be alert at all times to the position of the slide and hairline and look for the most direct means of obtaining the desired result.

.....

WHAT HAVE YOU LEARNED?

- 1...In the series circuit of Fig. 36, $R = 25\ \Omega$, and $X = 80\ \Omega$. What is the impedance Z ? What is the power factor of this circuit?

- 2... If the voltage supply E of a circuit similar to Problem 1 is 220 volts, the current I is 1 ampere and the power factor is 0.72, what is the value of Z , R , and X ? What is the power delivered to the circuit?
- 3... The circuit of Fig. 36 is to have an impedance of 1,200 Ω when R is 800 Ω . What must the value of X be? What is the power factor?
- 4... The circuit of Fig. 36 is to have an impedance of 625 Ω when $X = 157 \Omega$. What must the value of R be? If the circuit power is 200 watts, what is the value of E and I ?
- 5... In Fig. 36, if the impedance is 836 Ω and the power factor is 0.350, what is the value of R , X , and Θ ?
- 6... How much power is delivered to the circuit of Fig. 36 if $I = .52$ amperes, $R = 78.6 \Omega$, and $X = 28.9 \Omega$? What is the impedance and phase angle of this circuit?
- 7... A 110 volt electric heater has a resistance of 8.3 Ω and therefore draws 12.25 amperes when in operation. It is desired to operate this heater from a 220 volt line. What reactance must be put in series with this heater so that it only draws 12.25 amperes from the 220 volt line?

ANSWERS

- 1... $Z = 83.8 \Omega$, $P_f = .2982$
- 2... $Z = 220 \Omega$, $P = 158.4$ watts, $R = 158.4 \Omega$, $X = 152.8 \Omega$
- 3... $P_f = .667$, $X = 895 \Omega$
- 4... $R = 605 \Omega$, $E = 359.5$ volts, $I = .575$ amperes
- 5... $R = 292.5 \Omega$, $\Theta = 69.5^\circ$, $X = 782.5 \Omega$
- 6... $Z = 83.7 \Omega$, $\Theta = 20.2^\circ$, $P = 22.6$
- 7... $X = 14.4 \Omega$

USING SCALE L... Scale L is used in conjunction with scale D to find the mantissa of logarithms to the base 10, and conversely to find antilogarithms. To find the mantissa of the logarithm of a number, place the hairline over that number on scale D and then read the mantissa of the logarithm under the hairline on scale L. Thus, using this method we find the mantissa of the logarithm of 64.5 to be 0.809. The characteristic will be 1 in accordance with the usual rules for finding the characteristic. Hence, $\log 64.5 = 1.809$.

Finding antilogarithms is, of course, just the reverse of finding logarithms. To find the number whose logarithm is 4.4152 we set the hairline over the mantissa, 0.415, on scale L and then read the digits of the logarithm, 260, on scale D. Locating the decimal point in accordance with the rules of logarithms gives us $\text{antilog } 4.415 = 26,000$.

.....

Example...54

The output of an amplifier is 12 watts. The input to the amplifier is 0.032 watts. Find the gain of the amplifier in decibels.

Solution...

- (1) $\text{db} = 10 \log \frac{P_1}{P_2} = 10 \log \frac{12}{0.032}$
- (2) Set the left index of scale C opposite 12 on scale D.
- (3) Move the indicator so that the hairline is over 32 on scale CI.
- (4) Under scale L read 0.574. This is the mantissa of the logarithm.
- (5) Since $12/0.032$ is roughly 400, the characteristic of the logarithm will be 2.
- (6) $10 \times 2.574 = 25.74 \text{ db}$, the answer.

Explanation...

In step (1) the proper formula to use is obtained from the back of the Electronics slide rule. In steps (2) and (3) 12 is divided by 0.032 by multiplying 12 by the reciprocal of 0.032 (see Topic 18), using the CI scale. The reason for multiplying in this manner is so that the hairline is over the quotient on the D scale, and consequently over the logarithm of the quotient on the L scale. In this manner the required mantissa can be read directly on the L scale without the necessity of first reading the quotient on scale D.

WHAT HAVE YOU LEARNED?

Find the logarithm of:

1...427.6

3...25,000

5...0.218

2...4.276

4...0.00915

6...7

Find the antilog of:

7...2.437

8...0.815

9... $\bar{4}$.377

10... $\bar{1}$.444

11...The output from an amplifier is 25 watts, and the input to the amplifier is 0.025 watts. What is the gain of the amplifier in db?

12...Find the cube root of 365.2. (Hint: Divide the logarithm of 365.2 by 3 and take the antilogarithm of this result.)

ANSWERS

1. 2.631

5. $\bar{1}$.338

9. 0.000238

2. 0.631

6. 0.845

10. 0.278

3. 4.398

7. 273.6

11. 30 db

4. $\bar{3}$.961

8. 6.531

12. 7.148

SUMMARY

The following table summarizes the application of the operational methods explained in the manual to typical formulas in electronics. The text pages referred to in the table give detailed explanations of the procedures and also examples or practice problems.

The method given in the table for solving each formula is one of the shortest methods. In general there are many methods of working any problem on a slide rule, but some methods

are much more convenient than others. Every time a quantity is set up on a slide rule, read from the rule, or transferred to another scale, an error can possibly be made. Hence, the use of two operations where one is sufficient is not the best practice.

A formula not listed can be handled efficiently by locating in the table one equivalent in form and then solving by the method for the equivalent formula.

In using the following methods it is assumed that the slide will be reset (see page 29) if this operation should become necessary in order to continue with the method.

SUMMARY OF SETTINGS FOR COMMON FORMULAS

Formula	How to Solve	Explained in
$G = \frac{1}{R}$	(1) Opposite R on scale C, read G on scale CI.	Topic 15
$R_t = \frac{R_1 R_2}{R_1 + R_2}$	(1) Opposite R_1 on scale D, set the sum, $R_1 + R_2$ (found mentally), on scale C. (2) Opposite R_2 on scale C, read R on scale D.	Topic 16
$P = I^2 R$	(1) Set index of slide opposite I on scale D. (2) Opposite the resistance R on scale B, read P on scale A.	Topic 22
$I = \sqrt{\frac{P}{R}}$	(1) Set P on scale A opposite R on scale B. (2) Opposite the slide index, read I on scale D. If answer is unreasonable, repeat step (1) except use the other half of scale B for setting R.	Topic 23

Formula	How to Solve	Explained on Page
$R = \frac{P}{I^2}$	(1) Set slide index opposite I on scale D. (2) Opposite P on scale A, read R on scale B.	44
$P = \frac{E^2}{R}$	(1) Opposite E on scale D, set R on scale B. (2) Read P opposite the slide index on scale A.	44
$E = \sqrt{PR}$	(1) Set index of scale B opposite P on scale A. (2) Opposite R on scale A, read E on scale D. If result is unreasonable, repeat step (1) except use other half of scale A for setting R.	47
$pf = \frac{P}{EI}$	(1) Opposite P on scale D, set E on scale C. (2) Opposite I on scale CI, read pf on scale D.	32
$f = \frac{1}{2\pi\sqrt{LC}}$	(1) Determine approximate value of f from Decimal Point Locator scales. (2) Opposite L on scale H, set C on scale B. (3) Opposite the slide index, read f on scale D. If not in agreement with approximate value, repeat step (2) except use the other half of scale B for setting C.	64-67
$L = \frac{1}{(2\pi f)^2 C}$	(1) Set index of scale C opposite f on scale D. (2) Then opposite C on scale B, read L on scale H. (3) Find decimal point from Decimal Point Locator scales.	64-66

Formula	How to Solve	Explained on Page
$C = \frac{1}{(2\pi f)^2 L}$	(1) Set index of scale C opposite f on scale D. (2) Then opposite L on scale H, read C on scale B. (3) Find decimal point from Decimal Point Locator scales.	66
$X_L = 2\pi fL$	(1) Opposite f on scale 2π , set L on scale CI. (2) Opposite the slide index, read X_L on scale D. (3) Find decimal point from Decimal Point Locator scales.	60-61
$f = \frac{X_L}{2\pi L}$	(1) Set index of slide opposite X_L on scale D. (2) Opposite L on scale CI, read f on scale 2π . (3) Find decimal point from Decimal Point Locator scales.	60-64
$L = \frac{X_L}{2\pi f}$	(1) Set index of slide opposite X_L on scale D. (2) Opposite f on scale 2π , read L on scale CI.	60-64
$X_C = \frac{1}{2\pi fC}$	(1) Opposite f on scale 2π , set C on scale CI. (2) Opposite scale D index, read X_C on scale C. (3) Find decimal point from Decimal Point Locator scales.	60-64
$f = \frac{1}{2\pi CX_C}$	(1) Opposite index of scale D, set X_C on scale C. (2) Opposite C on scale CI, read f on scale 2π . (3) Find decimal point from Decimal Point Locator scales.	60-64

Formula	How to Solve	Explained on Page
$C = \frac{1}{2\pi f X_C}$	(1) Opposite index of scale D, set X_C on scale C. (2) Opposite f on scale 2π , read C on scale CI.	60-64
$P = IE \cos \Theta$	(1) Set E on scale CI opposite angle $(90 - \Theta)$ on scale S. (2) Opposite I on scale C, read P on scale D.	98
$\omega = 2\pi f$	(1) Read ω on scale D opposite f on scale 2π .	60
$Q = \frac{2\pi f L}{R}$	(1) Set R on scale C opposite f on scale 2π . (2) Opposite L on scale C, read Q on scale D.	60-61
$db = 10 \log \frac{P_1}{P_2}$	(1) Set index of slide opposite P_1 . (2) Opposite P_2 on scale CI, read mantissa of logarithm on scale L. (3) Add characteristic to mantissa and multiply by 10 to obtain db.	100-101
$Z = \sqrt{R^2 + X^2}$	(1) Set index of slide opposite R or X (whichever is larger) on scale D. (2) Set hairline over R or X (whichever is smaller) on scale D. (3) Read the black scale of T. (4) Move slide so that angle read in (3) is under hairline on black scale S. (5) Opposite scale C index read Z on scale D.	97-100
$R = \sqrt{Z^2 - X^2}$	(1) Set index at slide opposite Z on scale D. (2) Set the hairline over X on scale D. (3) Read the black scale of S. (4) Move slide so that angle read in (3) is under the hairline on black scale T. (5) Opposite scale C index read R on scale D.	97-100

Formula	How to Solve	Explained on Page
$\Theta = \sin^{-1} \frac{X}{Z}$	(1) Set index of slide opposite Z on scale D. (2) Opposite X on scale D, read Θ on black scale S.	97-100
$\Theta = \cos^{-1} \frac{R}{Z}$	(1) Set index of slide opposite Z on scale D. (2) Opposite R on scale D, read the angle on red scale S.	97-100

The examination for this lesson can be found immediately following the Appendix.



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PART 1

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A chat with your instructor

The Ln scale is used in place of the LL scales found on log-log type slide rules. The scale provides a method for solving engineering problems involving exponents, which are frequently encountered in electronics. Some examples of the type of practical problems solved with the Ln scale are given on page 16 of this Appendix.

Specifically, the Ln scale is a scale of natural logarithms, in contrast to the L scale which is a scale of common logarithms. Common logarithms, are exponents to base 10. For example, in the expression $10^{2.539} = 346$, 10 is called the base, and the exponent 2.539 is called the logarithm of 346, so that the expression can be written equally well: $\log_{10} 346 = 2.539$.

Any number could be used as the base for a system of logarithms. However, the only base besides 10 that is actually used for practical purposes is 2.71828, represented by the symbol e . When e is used as the base, the logarithms are called natural logarithms. The name comes from the fact that the value e frequently comes up in problems relating to natural phenomena. For example, if you roll a snow ball down a hill, it will gather additional snow rather slowly at first, but faster and faster as the ball becomes larger. The weight of the ball at any time can be roughly figured by the formula, $M = ke^{nt}$, where e is 2.71828, t is the time that the ball has been rolling and n and k are constants to make the weight come out correctly in pounds. The derivation of the value 2.71828 is beyond the scope of this training manual, but it can be found in any calculus text.

Working with natural logarithms follows the same general principles as with common logarithms. However, the characteristic of a natural logarithm is found in a different way than for common logarithms, and has a rather different meaning. The method and meaning are explained in the pages that follow.

In the following discussion you will occasionally find reference to a DI scale. Since your rule does not have this scale, you should use the CI scale instead. Where the instructions say to read on the DI scale, merely adjust the slide so that the C and D indexes coincide, and then read on the CI scale.

Since the Ln scale is intended primarily for the use of engineers, you may prefer to omit this section. In that case, your slide rule training course is now completed. We have made the training thorough, so that your Electronics Slide Rule can be your lifetime companion, to help you in your career in electronics.

The Ln scale is similar to the regular L scale. It is used for problems with base e . For many problems it is more convenient to use Ln than the Log Log scales on advanced models. In particular, it enables you to solve problems with *powers of e* in combined operations. Its range (from 0 to 2.3) is greater than the range of the L scale (from 0 to 1). The inclusion of the Ln scale completes the DUAL-BASE features of the Pickett Slide Rules.

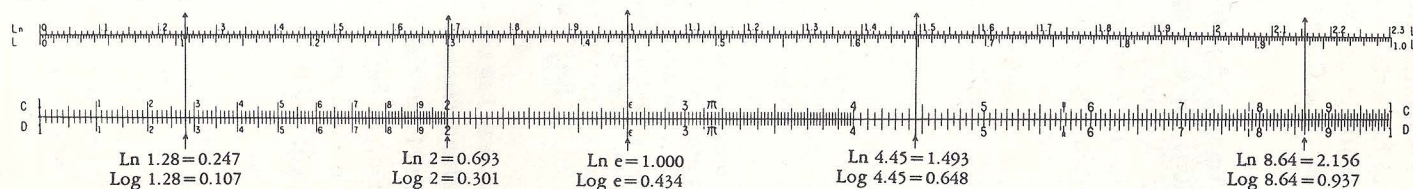
By computing a "characteristic" you can use the Ln scale to find any power of e ; thus the effective range for powers of e is from 0 to infinity. Since powers of e are read on the C (or D) scale, accuracy to 3 or 4 significant figures is obtained no matter how large or how small the numbers are. The Ln scale saves steps in many computational problems.

PART 1

In Part 1 you will learn how to use the Ln scale to find powers of e and logarithms to base e . The Ln scale is a uniformly divided scale similar to the L scale. It is used with problems in base e just as the L scale is used with problems in base 10.

When you study the charts in this manual always set the slide and cursor on your rule like the ones pictured in the manual. Study the *slide rule* itself along with the charts. In this manual we will use the symbol $\text{Ln } x$ for the logarithm of x to base e .

(a) Finding logarithms.



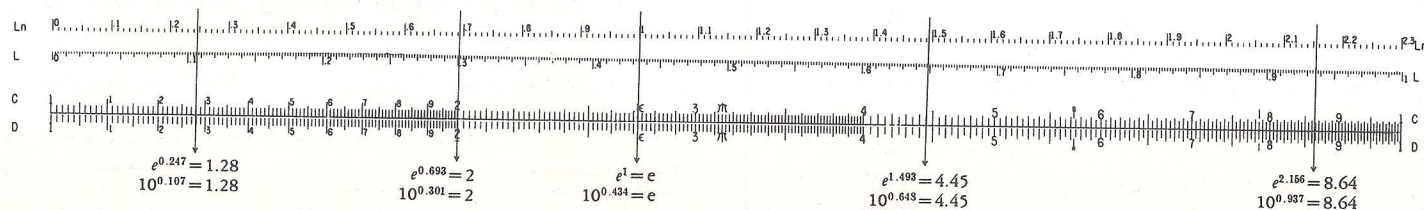
Rule: To find mantissas of logarithms: Set the number on the C (or D) scale, and read the mantissa for base 10 from L, or for base e from Ln. If L scales are on slide, set on C. If L scales are on body, set on D. Characteristic for base 10 is found by usual method. "Characteristic" for base e is explained in Section (e), page 8. Also, see below.

For $1 \leq x \leq 10$, we have $0 \leq \log x \leq 1$.
 For $1 \leq x \leq 10$, we have $0 \leq \text{Ln } x \leq 2.30258$
 For the same domain ($1 \leq x \leq 10$), the range of Ln is greater than the range of L.

Examples for practice. Verify that:

1. $\text{Ln } 1.473 = 0.387$
2. $\text{Log } 1.473 = 0.168$
3. $\text{Ln } 2.34 = 0.850$
4. $\text{Log } 2.34 = 0.369$
5. $\text{Ln } \pi = 1.145$
6. $\text{Ln } 3.49 = 1.250$
7. $\text{Ln } 4 = 1.386$
8. $\text{Ln } 4.62 = 1.530$
9. $\text{Ln } 5.17 = 1.643$
10. $\text{Ln } 7.35 = 1.995$
11. $\text{Ln } 8.05 = 2.086$
12. $\text{Ln } 9 = 2.197$
13. $\text{Ln } 3.62 = 1.286$
14. $\text{Log } 3.62 = 0.559$
15. $\text{Ln } 1.91 = 0.647$
16. $\text{Log } 1.91 = 0.281$
17. $\text{Ln } 2.66 = 0.978$
18. $\text{Log } 2.66 = 0.425$

(b) *Powers of e and of 10.* In the figure below, notice that the cursor hairlines shown are in the same positions as in Section (a) on page 4, above.



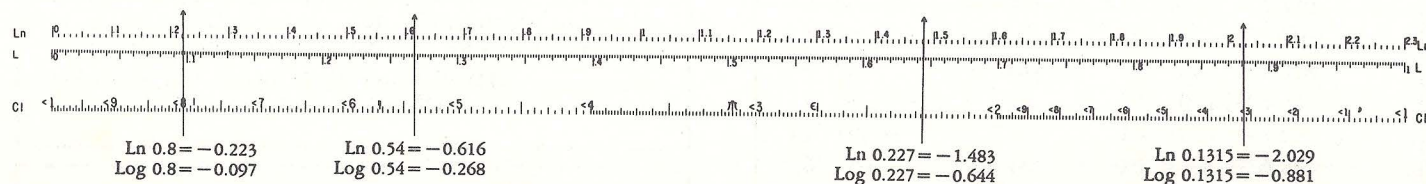
Rule: To find powers of e and of 10. Set the exponent of e on Ln, or of 10 on L, and read the power on C (or D). If L scales are on slide, use C; if they are on body, use D. The decimal point of the answer is found by special rules. See Section (f), page 9. Also see below.

For $0 \leq y \leq 1$, we have $1 \leq 10^y \leq 10$.
 For $0 \leq y \leq 2.30258$, we have $1 \leq e^y \leq 10$.
 Although the domain of y is greater for base e , the range of 10^y and e^y is the same. The exponents for this range may be set directly on the Ln or L scale.

Examples for practice. Verify that:

- | | |
|-----------------------------------|-------------------------|
| 1. $\sqrt{e} = e^{0.5} = 1.649$ | 2. $e^{0.81} = 2.25$ |
| 3. $\sqrt{10} = 10^{0.5} = 3.16$ | 4. $e^{1.15} = 3.16$ |
| 5. $e^{1.36} = 3.90$ | 6. $e^{1.63} = 5.104$ |
| 7. $e^{1.875} = 6.52$ | 8. $e^2 = 7.39$ |
| 9. $e^{2.138} = 8.48$ | 10. $e^{2.252} = 9.51$ |
| 11. $10^{0.732} = 5.40$ | 12. $10^{0.9} = 7.95$ |
| 13. $\sqrt{e^3} = e^{1.5} = 4.48$ | 14. $10^{0.405} = 2.54$ |
| 15. $e^{0.1} = 1.105$ | 16. $e^{0.205} = 1.228$ |
| 17. $e^{0.532} = 1.702$ | 18. $e^{0.01} = 1.0101$ |
| 19. $e^{0.99} = 2.691$ | 20. $e^{1.01} = 2.746$ |

(c) *Finding logarithms of proper fractions.* The logarithm of each proper fraction is a negative number. It is written in two ways; for example, $\log 0.5 = -0.301$ or $9.699-10$; also, $\ln 0.5 = -0.693$ or $9.307-10$. For slide rule work the form $9. _ _ -10$ is not needed.



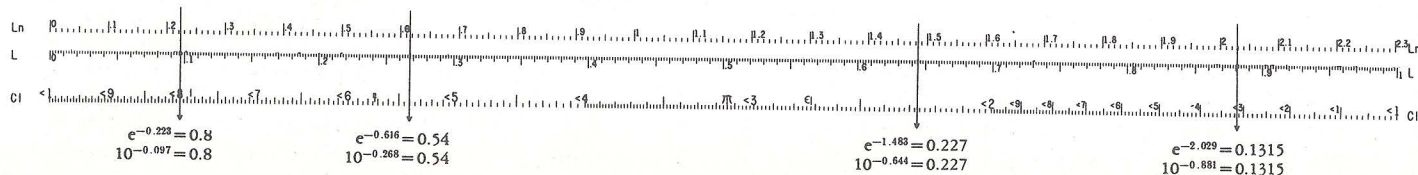
Rule: To find mantissas of logarithms of numbers between 0.1 and 1, set number on CI (or DI), read mantissa for base 10 on L, for base e on Ln. If L scales are on the slide, use CI. If L scales are on the body, use DI. For smaller numbers, see Section (f), page 9. Also, see below.

For $0.1 \leq x \leq 1$, we have $-1 \leq \log x \leq 0$.
 For $0.1 \leq x \leq 1$, we have $-2.30258 \leq \ln x \leq 0$.
 For the same domain ($0.1 \leq x \leq 1$), the range of Ln is greater than the range of L. For x in this domain, the logarithm is read *directly* from the scale and written with the negative sign. For x not in this domain, the characteristic must be found by a special rule.

Examples for practice. Verify that:

- | | |
|---------------------------|---------------------------|
| 1. $\ln 0.15 = -1.897$ | 2. $\ln 0.1625 = -1.818$ |
| 3. $\ln 0.19 = -1.661$ | 4. $\log 0.19 = -0.721$ |
| 5. $\ln 0.202 = -1.599$ | 6. $\log 0.202 = -0.695$ |
| 7. $\ln 0.259 = -1.351$ | 8. $\log 0.259 = -0.587$ |
| 9. $\ln 0.3 = -1.204$ | 10. $\ln \pi/10 = -1.158$ |
| 11. $\ln 0.85 = -0.163$ | 12. $\ln 0.92 = -0.083$ |
| 13. $\log 0.742 = -0.130$ | 14. $\ln 0.742 = -0.298$ |
| 15. $\log 0.363 = -0.440$ | 16. $\ln 0.363 = -1.014$ |
| 17. $\log 0.178 = -0.750$ | 18. $\ln 0.178 = -1.726$ |
| 19. $\ln 0.120 = -2.120$ | 20. $\log 0.12 = -0.921$ |
| 21. $\ln 0.103 = -2.27$ | 22. $\log 0.103 = -2.274$ |

(d) *Powers for negative exponents.* For negative exponents powers are all less than 1. Hence they are proper fractions. In the figure below, notice that the cursor hairlines are shown in the same positions as in Section (c) on page 6, above.



Rule: To find powers of e and of 10 for negative exponents, set the exponent of e on Ln, or the exponent of 10 on L, and read the power on CI (or DI). If L scales are on slide, use CI; if they are on the body, use DI. The decimal point in the answer is found by special rules. See Section (f), page 9. Also, see below.

For $-1 \leq y \leq 0$, we have $0.1 \leq 10^y \leq 1.0$.

For $-2.30258 \leq y \leq 0$, we have $0.1 \leq e^y \leq 1.0$.

Although the domain of y is greater for base e , the range of 10^y and of e^y is the same. The exponents for this range may be set directly on the Ln or the L scale.

Examples for practice. Verify that:

1. $e^{-2} = 0.135$
2. $10^{-0.8} = 0.1585$
3. $e^{-1} = 0.368$
4. $10^{-0.1} = 0.794$
5. $e^{-0.2} = 0.819$
6. $e^{-0.63} = 0.533$
7. $e^{-0.27} = 0.763$
8. $e^{-1.54} = 0.214$
9. $e^{-1.27} = 0.281$
10. $e^{-2.08} = 0.125$
11. $e^{-2.29} = 0.1013$
12. $e^{-1.65} = 0.192$
13. $10^{-0.87} = 0.2138$
14. $e^{-0.72} = 0.487$
15. $10^{-0.44} = 0.363$
16. $e^{-0.56} = 0.571$
17. $10^{-0.25} = 0.562$
18. $e^{-0.99} = 0.372$
19. $e^{-1.5} = 0.223$
20. $10^{-0.15} = 0.708$

(e) Finding the Characteristic

Base 10

The characteristic is the exponent of 10 when the number is expressed in *standard form*.

Rule. To express a number in standard form: (i) place a decimal point at the right of the first non-zero digit, (ii) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is positive (+). If the original decimal point is toward the left, the characteristic is negative (−). Indicate that the result of (i) is multiplied by 10 with this exponent.

Examples for practice

Number	Number in standard form	Characteristic
1. 5,790,000	5.79×10^6	6
2. 0.000283	2.83×10^{-4}	−4
3. 44	4.4×10^1	1
4. 0.623	6.23×10^{-1}	−1
5. 8.15	8.15×10^0	0
6. 461,328	4.61328×10^5	5
7. 0.0000005371	5.371×10^{-7}	−7
8. 0.0306	3.06×10^{-2}	−2

Base *e*

The term “characteristic” as used here will mean the number to which a reading from the Ln scale must be added to account for logarithms not in its range.

Rule: First express the number in standard form.

Read the logarithm of the first factor directly from Ln, as in Section (a), page 4. Multiply 2.30258 by the exponent of 10 in the second factor. If the exponent is positive, add this result to the direct reading. If the exponent is negative, subtract the result from the direct reading.

Examples for practice. Verify that the logarithm to base *e* for the examples at the left is as follows:

1. $\text{Ln } 5.79 \times 10^6 = 1.756 + 6(2.303) = 15.574$
2. $\text{Ln } 2.83 \times 10^{-4} = 1.040 - 4(2.303) = -8.170$
3. $\text{Ln } 4.4 \times 10^1 = 1.482 + 2.303 = 3.785$
4. $\text{Ln } 6.23 \times 10^{-1} = 1.829 - 2.303 = -0.474$
5. $\text{Ln } 8.15 \times 10^0 = 2.098 - 0 = 2.098$
6. $\text{Ln } 4.61 \times 10^5 = 1.528 + 5(2.303) = 13.041$
7. $\text{Ln } 5.371 \times 10^{-7} = 1.681 - 7(2.303) = -14.437$
8. $\text{Ln } 3.06 \times 10^{-2} = 1.118 - 2(2.303) = -3.487$

For $0 < x < \infty$, we have $-\infty < \log x < +\infty$.

For $0 < x < \infty$, we have $-\infty < \text{Ln } x < +\infty$.

(f) *Extending the range for 10^y and e^y .*

Base 10

For y not in the interval between 0 and 1, the standard method of finding 10^y first expresses it as the product of two factors. Thus $10^{2.5} = 10^2 \times 10^{0.5}$. One factor has an integral exponent. The other factor has a fractional exponent in the interval 0 to 1. The second factor is computed by the methods of Section (b), page 5, and Section (d), page 7. The first factor then determines the position of the decimal point in the final answer.

Examples for study.

1. $10^{2.5} = 10^2 \times 10^{0.5} = 10^2 \times 3.16 = 316.$
2. $10^{4.26} = 10^4 \times 10^{0.26} = 10^4 \times 1.82 = 18,200.$
3. $10^{-5.38} = 10^{-5} \times 10^{-0.38} = 10^{-5} \times 0.417 = 0.00000417.$
4. $10^{-2.71} = 10^{-2} \times 10^{-0.71} = 10^{-2} \times 0.195 = 0.00195.$

For $-\infty < y < +\infty$, we have $0 < 10^y < +\infty$.
The effective range for powers of 10 is infinite.
Three or four significant figures of e^y can be found.

Examples for practice. Verify that:

1. $10^{3.916} = 8,240.$
2. $10^{-3.916} = 0.0001214$
3. $10^{5.023} = 105,400.$
4. $10^{-5.023} = 0.000,009,48$
5. $10^{14.622} = 4.19 \times 10 = 419,000,000,000,000.$
6. $10^{-23.877} = 0.000,000,000,000,000,000,001,327$

Base e

For y not in the interval $0 \leq y \leq 2.30258$, express e^y as the product of two factors. For example, $e^{3.5} = e^{2.3} \times e^{1.2} = 10e^{1.2}$. One method of finding these factors is to divide the exponent y by 2.30258 (or a rounded value of this divisor, such as 2.303), to determine an integral quotient, q , and a remainder, r . Then $y = 2.303q + r$, and $e^y = e^{2.303q+r} = e^{2.303q} \times e^r = (e^{2.303})^q \times e^r = 10^q \times e^r$.

The value of the second factor is computed by the methods of Section (b), page 5, and Section (d), page 7. The first factor is used to determine the decimal point.

Examples for study.

1. To find $e^{6.54}$, first divide 6.54 by 2.303, obtaining quotient 2 and remainder 1.934. Then $e^{6.54} = 10^2 \times e^{1.934}$. Set cursor to 1.934 on Ln. Read 6.92 on C (or D). Then answer is $100 \times 6.92 = 692.$
2. Find $e^{-6.54}$. As in Example 1, $e^{-6.54} = 10^{-2} \times e^{-1.934}$. Set cursor hairline to 1.934 on Ln. Read 0.114 on CI (or DI). Then $e^{-6.54} = 0.00144.$
3. Find $e^{17.4}$. Divide 17.4 by 2.303, obtaining quotient ("characteristic") 7 and remainder 1.28. Set hairline to 1.28 of Ln. Read 360 on C (or D). Multiply by 10^7 .

For $-\infty < y < +\infty$, we have $0 < e^y < +\infty$. The effective range for powers of e is infinite. Three or four significant figures of e^y can be found.

(g) *A short cut in using $\text{Ln}10 = 2.30258$.*

To extend the range of Ln the number 2.30258 is needed. Suppose that, to save work, the number 2.3 is used. Some error will of course occur. For example, the remainder in division will be too large. How can we easily correct for this error? The following simple rule will serve:

Rule. Take 1 percent of the quotient and divide it by 4. Subtract the result from the remainder to obtain the correct remainder to set on Ln.

Example. Find $e^{17.4}$ (Compare with Example 3, page 9, under Base e). Divide:

$$\begin{array}{r} 7. \\ 2.3 \overline{) 17.4} \quad \text{or} \quad 2.30258 \overline{) 17.40000} \\ \underline{16.1} \\ 1.3 \end{array} \quad \begin{array}{r} 7. \\ \underline{16.11806} \\ 1.28194 \end{array}$$

Take 1% of 7; $0.01 \times 7 = 0.07$; Divide by 4. $0.07 \div 4 = 0.02$, approx. Subtract 0.02 from 1.3, to obtain 1.28, the corrected remainder. Then $e^{17.4} = e^{1.28} \times 10^7$. The basis of this rule is explained below.

Consider $x = e^n$. Divide n by 2.30258, and denote the integral part of the quotient by q and the remainder by r . Then, $n = 2.30258q + r$, $r < 2.30258$.

We now propose to use 2.3 as divisor in place of 2.30258. We require the quotient to again be q , but get a new remainder which we denote by R , where $R > r$. Then,

$$n = 2.3q + R, \quad \text{where } R < 2.3.$$

Subtracting this from the former equation, we have $0 = 0.00258q + r - R$, or $r = R - 0.00258q$.

Thus the error, $R - r$, in the remainder is $0.00258q$. If this is rounded off to $0.0025q$, it expresses one-fourth of 1 percent of the quotient.

When the slide rule is used to divide by 2.30, proceed as follows: Set 2.30 of C over 17.4 of D. Under 1 of C read 7.56 on D. The integral part, or "characteristic", is 7. Multiply the decimal fraction 0.56 by 2.3, using the C and D scales. Obtain 1.29 as the reduced exponent of e .

With Model 4 rules the quotient may be obtained by merely setting the exponent on DF/M and reading the quotient on D. The relation between readings on the D and the DF/M scales may be indicated symbolically as follows:

$$(D) \times 2.30 = (DF/M) \quad \text{and} \quad (DF/M) \div 2.30 = (D).$$

For some purposes and for some exponents, this slide rule method is not sufficiently accurate.

Examples for practice.

1. Find $e^{7.61}$. Divide 7.61 by 2.3; quotient 3, remainder 0.71. Correction is $0.03/4 = .01$. Hence $e^{7.61} = e^{0.70} \times 10^3$, or 2,018.
2. Find $e^{-6.95}$. Divide 6.95 by 2.3 to get characteristic 3 and remainder 0.05. Correction is $0.03/4 = 0.0075$. Corrected remainder is 0.0425. Hence $e^{-6.95} = e^{-0.0425} \times 10^{-3}$. Set 0.0425 on Ln, read 0.958 on CI. Point off 3 places to the left, to get 0.000958.
3. Find e^9 . Divide 9 by 2.3. Characteristic is 3, remainder 2.1. Correction is $0.03/4 = 0.0075$, or .01. $e^9 = e^{2.09} \times 10^3$. Set 2.09 on Ln, read 8.1 on C. Then $e^9 = 8100$.
4. Find e^{-9} , or $e^{-2.09} \times 10^{-3}$. Set 2.09 on Ln. Read on 0.125 on CI, point off 3 places to left to find $e^{-9} = 0.000123$.

PART 2

In Part 2 you will learn how the L and Ln scales are used in combination with other scales. The methods used when the L scales are on the body differ from those used when they are on the slide. Follow only the instructions for the type of slide rule you have.

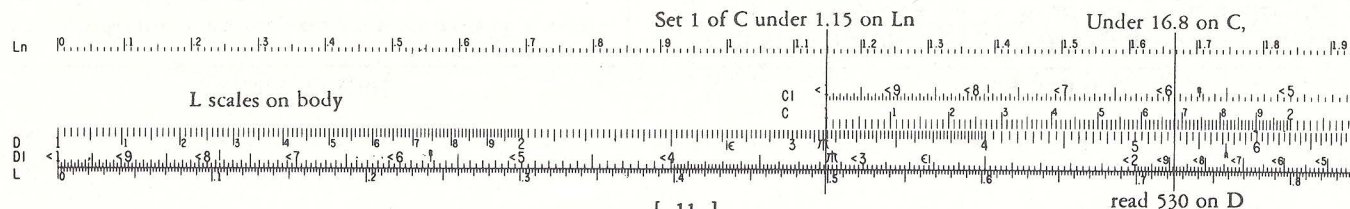
In solving problems, first express the numbers in standard form as explained in Section (f) and Section (g), pages 9 and 10. The calculations are carried through within the ranges of L and Ln provided, and the decimal points are determined by special rules.

(b) *Multiplication with powers.* The scales below are set to find $16.8 \times e^{1.15}$ with L scales on the slide.



Notice that when 1 of the C scale is set over 16.8 of the D scale, the product of 16.8 and any number set on C is read on D. But by setting the cursor hairline over 1.15 of Ln the value of $e^{1.15}$ is *automatically* set on C. This number (actually 3.16) does not have to be read. The product is on D. With the log log scales, this value (3.16) must be read and transferred to C before the multiplication can be started.

Rule for $a \cdot e^v$. If L scales are on slide, set 1 of C over a on D. Move cursor hairline to y of Ln. Read figures of answer on D. Determine the decimal point by standard form method. If L scales are on body, *begin* with e^v . Set cursor hairline to y on L. Set 1 of C under cursor hairline. Move cursor to a of C. Read answer on D. With powers of 10, use L in the same way.



- (i) **Division with powers.** Remember that division is the opposite of multiplication. The scales pictured in Section (h), on page 11, are set to divide 530 by $e^{1.15}$; that is, to find $530/e^{1.15}$ using D and C, or $530 e^{-1.15}$, using D and CI.

L scales on slide

Rule: To divide a/e^y , set y on Ln over a on D. Under 1 of C read a/e^y on D.

(j) Examples for practice.

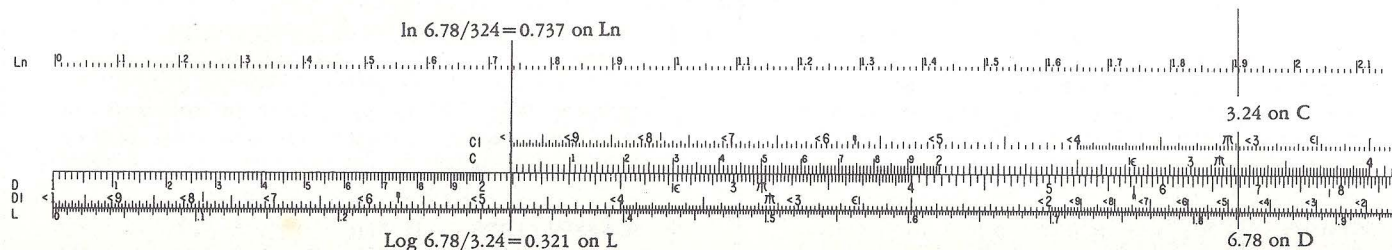
- Find $2.79 e^{1.945}/3.82$. Set hairline over 2.79 on D. Move slide so 3.82 on C is under hairline. Move hairline to 1.945 on Ln. Read 5.12 on D.
- Find $17.35 e^{1.226} \sin 43^\circ$. Set 1 of C over 17.35 on D. Move hairline over 1.226 on Ln. Move right index of C under hairline. Move hairline to 43 on S. Read 40.3 on D.
- Find $0.0000452 e^{7.61}$ (see Ex. 1, p. 10). Write the work in standard form:
 $4.52 \times 10^{-5} \times e^{0.70} \times 10^3 = 4.52 \times e^{0.70} \times 10^{-2}$. Set index of C over 452 on D. Move hairline over 0.70 on Ln. Read 912 on D. Answer is 0.0912.
- Find $5.27^2 e^{12.7}$. First rewrite $e^{12.7}$ as $e^{1.19} \times 10^5$. Set hairline over 5.27 on $\sqrt{\quad}$. Turn rule over, and set index of C under hairline. Move hairline to 1.19 on Ln. Read 910 on D. Note 5.27^2 is about 30 or, roughly, 3×10 . Also $e^{1.19}$ is about 3. Answer, then, is about $3 \times 3 \times 10 \times 10^5$ or 9×10^6 . Correct to three significant figures, answer is 9.10×10^6 .

L scales on body

Rule: To divide a/e^y , set 1 of C under y of Ln. Move hairline over a on D. Read a/e^y on C under the hairline.

- Find $2.79 e^{1.945}/3.82$. Set hairline over 1.945 of Ln. Set slide so 3.82 of C is under hairline. Move hairline over 2.79 on C. Read 5.12 on D.
- Find $17.35 e^{1.226} \sin 43^\circ$. Set hairline over 1.226 on Ln. Move slide so 17.35 on CI is under hairline. Move hairline to 43 on S. Read 40.3 on D.
- Find $0.0000452 e^{7.61}$ (see Ex. 1, p. 10). Write the work in standard form: $4.52 \times 10^{-5} \times e^{0.70} \times 10^3 = 4.52 \times e^{0.70} \times 10^{-2}$. Set hairline over 0.70 on Ln. Move slide so index of C is under hairline. Move hairline over 452 on C. Read 912 on D. Answer is 0.0912.
- Find $5.27^2 e^{12.7}$. First rewrite $e^{12.7}$ as $e^{1.19} \times 10^5$. Set hairline over 1.19 on Ln. It is not convenient to use the A scale for 5.27^2 . Move slide so 527 on CI is under hairline. Move hairline to 527 on C. Read 910 on D. Note 5.27^2 is about 30, or, roughly, 3×10 . Also $e^{1.19}$ is about 3. Answer, then, is about $3 \times 3 \times 10 \times 10^5$ or 9×10^6 . Correct to three significant figures, answer is 9.10×10^6 .

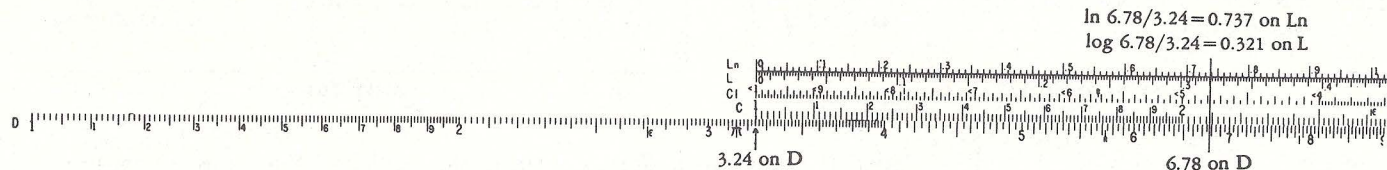
(k) *Logarithms of combined operations.* The scales below are set to find $\ln 6.78/3.24$ and $\log 6.78/3.24$ with L scales on the body.



First, divide 6.78 by 3.24 in the usual way using C and D scales. Move cursor over 1 of C. Read $\ln 6.78/3.24$ on Ln and read $\log 6.78/3.24$ on L. If L scales are on the slide, close rule (move slide so C and D indexes coincide) before reading from Ln or L. Or, if you prefer, first set slide so 1 of C is over 3.24 on D. Move hairline over 6.78 on D. Read logarithm from Ln or L.

Rule: To find $\ln a/b$ or $\log a/b$, set b on C over a on D. At index of C read $\ln$ on Ln and $\log$ on L. Characteristics must be found by a special rule.

If L scales are on slide, set 1 of C over b on D. Move indicator over a on D. Read logarithm from Ln on L. This method requires only one setting of the slide.



- (1) *Powers of other bases.* Sometimes powers of bases other than 10 or e are needed. If a slide rule has Log Log scales, they may be used to find these powers within the range provided. If the slide rule does not have Log Log scales, or if the power is outside the scale range provided, one of the following methods may be used.

Using base e .

Examples

- Find $1.5^{2.4}$. Write $e^x = 1.5$. Set 1.5 on C (or D), find $x = 0.405$ on Ln. Then $(1.5)^{2.4} = (e^{0.405})^{2.4} = e^{0.972}$ by multiplying the exponents. Using Ln again, set 0.972 on Ln, read the answer 2.65 on C (or D). This solution can be expressed in logarithmic form as follows: $\ln 1.5^{2.4} = 2.4 \ln 1.5$.
- Find $18.5^{-6.37}$. Set $y = 18.5^{-6.37}$. Then $\ln y = -6.37 \times \ln 18.5$. Now $\ln 18.5 = \ln 1.85 \times 10 = 0.615 + 2.303$ or 2.918. Now $-6.37 \times 2.918 = 18.58$ and $18.58 \div 2.3 = 8$ with remainder of 0.18. But 1% of 8 = .08, and $.08 \div 4 = .02$, so with the correction $0.18 - 0.02 = 0.16$, we have to find $e^{-0.16} \times 10^{-8}$. Set hairline over 0.16 on Ln, read 0.852 on CI (or DI). The quotient 8 tells us the answer is $0.852 \times 10^{-8} = 8.52 \times 10^{-9}$.
- Find $0.88^{0.25}$. Set $y = 0.88^{0.25}$. Then $\ln y = 0.25 \times \ln 0.88$. Write $\ln 0.88 = \ln 8.8 \times 10^{-1}$. Set hairline over 8.8 of C (or D), read 2.175 on Ln. Then $\ln 8.8 \times 10^{-1} = 2.175 - 2.303 = -0.128$, and $0.25 \times (-0.128) = -0.032$. Set hairline over 0.032 on Ln, read answer 0.968 on CI (or DI).

Using base 10.

Examples

- Find $1.5^{2.4}$. Write $10^x = 1.5$. Set 1.5 on C (or D), find $x = 0.176$ on L. Then $(1.5)^{2.4} = (10^{0.176})^{2.4} = 10^{0.422}$, by multiplying exponents. Set 0.422 on L, read the answer 2.65 on C (or D). This solution can be expressed in logarithmic form as follows: $\log 1.5^{2.4} = 2.4 \log 1.5$.
- Find $18.5^{-6.37}$. Set $y = 18.5^{-6.37}$. Then $\log y = -6.37 \times \log 18.5$. Now $\log 18.5 = 0.267 + 1 = 1.267$. Now $-6.37 \times 1.267 = -8.07$. We must now set the hairline over 0.07 on L, *reading from right to left*, or subtract 8.07 from 10.00 - 10 to write the logarithm with a positive mantissa, namely 1.93 - 10. Set the hairline over 0.93 on L and read 8.50 on C (or D). Then the result is 8.50×10^{-9} .
- Find $0.88^{0.25}$. Set $y = 0.88^{0.25}$. Then $\log y = 0.25 \times \log 0.88$. Write $\log 0.88 = \log 8.8 \times 10^{-1}$. Set hairline over 8.8 on C (or D), read 0.944 on L. Then $\log 8.8 \times 10^{-1} = 0.944 - 1$, or 3.944 - 4. $0.25(3.944 - 4) = 0.986 - 1$. Set hairline over 0.986 of L, read 9.68 on C (or D). Then answer is 0.968.

- (m) **Hyperbolic functions.** The Ln scale is very helpful in finding values of the hyperbolic functions. This is especially true for Model 1011 which does not provide Log Log scales or hyperbolic function scales. However, even with Model 4 on which these extra scales are available, the Ln scale simplifies the work in problems that fall outside the range of the scales provided.

By definition, $\sinh x = (e^x - e^{-x})/2$, or $\sinh x = (e^x/2) - (e^{-x}/2)$.

By definition, $\cosh x = (e^x + e^{-x})/2$, or $\cosh x = (e^x/2) + (e^{-x}/2)$.

By definition, $\tanh x = (e^x - e^{-x})/(e^x + e^{-x}) = (\sinh x)/(\cosh x)$.

Rule: To find $\sinh x$ or $\cosh x$, set hairline over x on Ln, read e^x on C (or D) and e^{-x} on CI (or DI). For $\sinh x$, subtract e^{-x} from e^x and divide the result by 2. For $\cosh x$, add e^x and e^{-x} , and divide by 2. To find $\tanh x$, use Ln to find e^x and e^{-x} ; divide their difference by their sum.

Rule: For $x > 3$, $\sinh x = \cosh x = e^x/2$ can be found by setting the index of the C scale over 5 on the D scale, moving the hairline to x on Ln, and reading the result on D.

Examples.

1. Find $\sinh 5.4$ or $e^{5.4}/2$. Divide 5.4 by 2.303, obtaining quotient 2 and remainder 0.794. $(e^{5.4}/2) = (e^{0.794}/2)10^2$. Set right hand index of C over 5 of D. Move hairline to 0.794 on Ln. Read 1.107 on D. This must be multiplied by 10^2 , so $\sinh 5.4 = 110.7$.
2. Find $\sinh 24 = e^{24}/2$. First write $e^{24} = e^{0.975} \times 10^{10}$. Using Ln, find $e^{0.975} = 2.65$. Then $2.65/2 = 1.32$, so $\sinh 24 = 1.32 \times 10^{10}$.

Extending the ranges. Remember that if $x > 2.3$, you must first divide x by 2.3 and correct the remainder. The corrected remainder is set on Ln instead of x , and the integral quotient q is the exponent of 10 such that the factor 10^q determines the position of the decimal point.

For $x > 3$, we have $(e^{-x}/2) < 0.025$. Hence, for $x > 3$, $\sinh x = \cosh x = e^x/2$, approximately, and $\tanh x = 1$.

For $x < 0.10$, we have $\sinh x = x$, $\cosh x = 1$, and $\tanh x = x$, approximately.

On Model 4 the values of e^x and e^{-x} for $x < 23$ can be found directly on the Log Log scales by using the DF/M scale. For $x > 10$ the accuracy is poor. For $x > 23$, the Log Log scales are useless.

Examples.

3. Find $\cosh 4.8 = e^{4.8}/2$. Write $e^{4.8} = e^{0.195} \times 10^2$. Set left index of C over 5 on D. Move hairline over 0.195 on Ln. Read $e^{0.195}/2 = 0.608$ on D. Then $\cosh 4.8 = 60.8$.
4. Find $\tanh 1.3$. Using Ln, read $e^{1.3} = 3.67$ on C (or D) and $e^{-1.3} = 0.273$ on CI (or DI). Then $3.67 - 0.273 = 3.397$ and $3.67 + 0.273 = 3.94$; hence $\tanh 1.3 = 0.862$.

(n) *Applied problems.*

1. As an extraordinary example consider the following quotation:
 "The total N for the entire line is $N = 0.1118 \times 2000 = 223.6$ nepers, and the ratio of input to output current

$$I_s/I_r = e^{223.6} \doteq 10^{97} \text{**}$$

Calculate $e^{223.6}$.

By the method of Section (g), page 10, above, we divide 223.6 by 2.3, and correct the remainder.

$\begin{array}{r} .97 \\ 23 \overline{) 223.60} \\ \underline{207} \\ 16.6 \\ \underline{16.1} \\ .50 \end{array}$	$\begin{aligned} 0.01 \times 97 &= 0.97 \\ 1/4 \times .97 &= 0.24 \\ 0.50 - .24 &= 0.26 \end{aligned}$
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Set hairline over 0.26 of Ln.

Read 1.297 on C.

The result is 1.297×10^{97} . The result found by logarithms is 1.286×10^{97} . The error is 0.85%, or under 1%, and occurs because the correction formula uses 0.0025 instead of 0.00258. This shows that the method using Ln is sufficiently accurate for all exponents up to 100; such large exponents are exceedingly rare.

*Ware, Lawrence A. and Reed, Henry R. *Communication Circuits*. (New York, 1942), John Wiley and Sons, Inc. p. 49.

2. A table of standard sizes for rectangular wire may be made by inserting 38 geometric means between the diameter (0.46 in.) of Gauge 0000 and the diameter (0.005 in.) of Gauge 36 of the American Wire Gauge.

Calculate the common ratio $r = \sqrt[39]{\frac{0.4600}{0.005}}$, and compute the 36th term.

First note that $r = (460/5)^{1/39}$, or $(92)^{1/39}$. Write $10^x = 92$. Set 92 on C (or D), find mantissa of x, or 0.964 on L. Then $x = 1.964$. Then $r = (10)^{1.964/39} = 10^{0.0503}$; set 0.0503 on L, read $r = 1.123$ on C (or D). The 36th term is 0.005×1.123^{35} , or $0.005 \times 10^{35} \times (0.0503) = 0.005 \times 10^{1.761}$. Set hairline over 0.761 on L, read 5.77 on C (or D). Finally, compute $0.005 \times 10 \times 5.77 = 0.289$ in., or 289 mils, approximately.

3. The formula for the current in a certain circuit is $i = 1.25 (1 - e^{-80t})$, $0 \leq t \leq 0.01$. Find i for $t = 0.006$; that is,
 $i = 1.25 (1 - e^{-80 \times 0.006}) = 1.25 (1 - e^{-0.48})$.
 Set hairline over 0.48 on Ln, read $e^{-0.48} = 0.619$ on CI (or DI). Then $i = 1.25 (1 - 0.619) = 1.25 \times 0.381 = 0.476$.
4. In a problem similar to 3, above, the formula is $i = 4 (1 - e^{-40t})$, $0 \leq t \leq 0.02$. Find i for $t = 0.015$; that is,
 $i = 4 (1 - e^{-0.60})$. Answer: 1.804.

LESSON 3125-1

SLIDE RULE

EXAMINATION

1. Given a right triangle in which $a = 86.7$ and $b = 49.8$, you can best find the value of angle A by
 1. setting the right index of scale C opposite 86.7 on scale D, and the hairline over 49.8 on scale D. Then read the answer 29.9° under the hairline on black scale T.
 2. proceeding as above, except read the answer, 60.1° , on red scale T.
 3. proceeding as in selection 1, except read the answer, 54.9° on red scale S.
 4. setting the left index of scale C opposite 49.8 on scale D, and the hairline over 86.7 on scale D. Then read the answer, 9.86° under the hairline on black scale T.
 5. proceeding as above, except read the answer, 81.4° , on red scale T.
2. Refer to the last question. Having found angle A, you can now best find the hypotenuse by
 1. using the formula $c^2 = a^2 + b^2$.
 2. with the position of the hairline unchanged from the last question, move slide until angle A on red scale S is under the hairline, and then read the answer opposite the C scale index on scale D.
 3. same as selection 2, except use black scale S.
 4. same as selection 2, except read answer on C scale opposite scale D index.
3. What scales are used to find the sine of 2.46° on the slide rule?
 1. The S and the C scales.
 2. The S and the CI scales.
 3. The CI and D scales.
 4. The C and D scales.
4. What is the sine of 2.46° ?

1. 0.0429	2. 0.0431	3. 0.0416
4. 0.429	5. 0.431	6. 0.434
5. What angle has a tangent of 0.0587?

1. 0.332°	2. 3.04°	3. 3.32°	7. 59.8°
4. 3.36°	5. 5.98°	6. 30.4°	

6. In solving right triangles with the slide rule you place the decimal points by remembering that within the range of angles on the S and T scales,
1. the legs are always shorter than the hypotenuse, but never shorter than one-fifth the length of the hypotenuse.
 2. the sum of the squares of the two legs is equal to the square of the hypotenuse.
 3. the legs are always shorter than the hypotenuse, and sum of the two legs are always greater than the hypotenuse.
 4. the hypotenuse is always longer than either leg, but never more than ten times the length of either leg.
7. Find the value of c in a right triangle if $a = 32$ and $A = 48.42^\circ$.
1. 28.4
 2. 26.3
 3. 26.6
 4. 42.8
 5. 44.4
8. In the triangle of the last question, find the value of b . (Select answer from choices for the last question)
9. Find the value of angle B in a right triangle in which $a = 3.04$ and $b = 2.51$.
1. 38.1°
 2. 39.5°
 3. 50.2°
 4. 50.4°
10. Find the impedance of circuit which has an inductance with a reactance of 358 ohms in series with a 455 ohm resistor.
1. 381 ohms
 2. 496 ohms
 3. 579 ohms
 4. 590 ohms
11. Referring to the last question, what will be the phase angle between voltage and current in that circuit?
1. 27.2°
 2. 38.2°
 3. 61.8°
 4. 62.8°
12. Referring to the last two questions, what is the power factor of the circuit?
1. 0.214
 2. 0.432
 3. 0.518
 4. 0.786
13. An audio amplifier has an input power of 0.25 watts and an output power of 2 watts. What is its db gain?
1. 4.23 db
 2. 8.0 db
 3. 9.0 db
 4. 18.0 db
 5. 18.4 db
14. The input power to a transmission line is 320 watts. The power output to the antenna is 270 watts. Find the transmission line loss in decibels.
1. 0.74 db
 2. 1.48 db
 3. 5.6 db
 4. 7.4 db
 5. 14.8 db

END OF EXAM



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