Usage Instructions

for the "Py-Lo" slide rule

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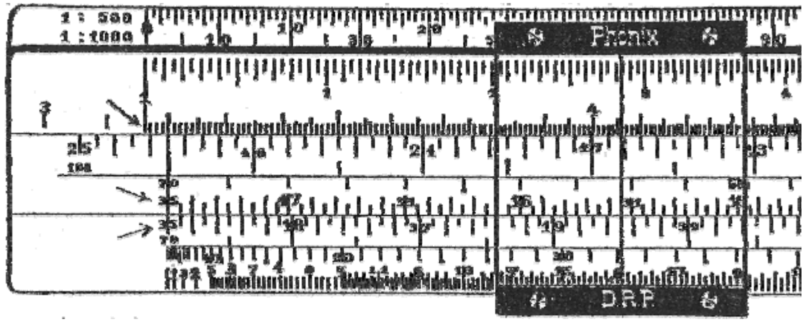


The present slide rule has been designed by the official land surveyor and railway bailiff Mr Seifert in Saarbrücken. It is primarily designed for the most common calculations in the geodetic practice.

Special importance has been given to achieve enough accuracy to meet the requirements needed in geodetic calculations. This is in part achieved by dividing the scales in the middle and placing one half over or below the other respectively.

The entire length of each scale is 625 mm. Therefore, the accuracy is considerably increased in comparison to the common slide rule with a length of 250 mm. It is fully enough for example in surface calculations by extrapolation, provided that these are decomposed in factors over 100,00.

Nevertheless, the instrument with the case may be easily placed in a briefcase, so that it can also be used in field work.

The instrument comprises two fundamentally different and not related scale systems, that are:

1. The logarithmic (Lo) scale system: the body upper part in combination with one side of the slide. Here the mantissas of the logarithms are marked to scale. It is used for multiplication, division, powers, and square roots.

2. The Pythagorean (Py) scale system: the body lower part in combination with the other side of the slide. Here the number squares are marked to scale. It is used to evaluate the Pythagorean formula in the simplest way, reading directly one side of a right triangle after one-time adjustment of the other two sides.

Consistently on both systems, the scales on the slide are arranged reversed, that is from right to left, while in the body, as usual, these go from left to right, and this must be taken into account when reading. This arrangement has the advantage that for the most common slide rule uses, like for multiplication and when calculating the hypotenuse from the smaller sides, misalignment can be avoided, and the result can always be read without further slide movement.

In order to simplify the explanations hereafter, the different scales have been identified with letters, like in the figures. On the instrument these letters do not exist.

1.- <u>THE "LO" SCALES</u>

It consists of the scales A and B on the body top strip, and C and D on the slide. Scale B is the continuation of the scale A, and scale D continues the scale C (see figures).

1.1.- <u>Multiplication</u>

Set the two factors with the help of the cursor vertically one above the other and read the result against the longer line marked by an arrow. It does not matter whether you set the first factor on the body and the second on the slide or, vice versa, the second on the body and the first on the slide. Both lead to the same position of the slide.

Then, follow this simple rule: If the two factors are in A and D or in B and C, then read at right from the cursor, and if they are in A and C or in B and D, then read at left from the cursor.

Generally speaking: If between the scales in which the factors are set there is another scale, read at cursor left by ↗ or ↘, if none or two scales are in between, then read at cursor right by ⤢ or ⤡. The rule is indicated similarly in the mentioned magazines. Examples:

a) 17,37 x 26,22 = 455,4   figure 1

Cursor to 17,37 on A scale, 26,22 in C under the cursor by moving the slide (at the black triangles ▲), or cursor (dotted line) to 26,22 in A (at ▲), 17,37 in C under the cursor (at ▲), read 455,4 left at ↗ in B.

b) 17,37 x 8,29 = 144,0   figure 1

Cursor to 17,37 in A (at △), and 8,29 in D under it (at ▽), or cursor to 8,29 in B (at ▽), and 17,37 in C under it (at △), read 144,0 right by ⤢ in C.

c) 7,08 x 32,25 = 228,3   figure 2

Cursor to 7,08 in B (at ▼), and 32,25 in D under it (at ▼), or cursor to 32,25 in B (at ▼), and 7,08 in D under it (at ▼), read 228,3 left by ↘ in C.

d) 7,08 x 102,0 = 722,2   figure 2

Cursor to 7,08 in B (at ▽), and 102,0 in C under it (at △), or cursor to 102,0 in A (at △), and 7,08 in D under it (at ▽), read 722,2 right by ⤡ in B.

1.2.- Multi-factor products can naturally be calculated without reading intermediate results

Example: 17,37 x 8,29 x 2,316 = 333,5

Calculate first 17,37 x 8,29 like in b). The intermediate result 144,0 at right by ⤢ is not read. Move the cursor left to the line by ↗ and so to 144,0 in A.

Once the number in A is selected, push 2,316 in C under the cursor line, and read, as between A and C there is only one scale, by ↗ in B 333,5.

A way to determine the position of the coma is not explained. This has not been found with previous slide rules. The number of digits can be easily determined by mental calculation.

1.3.- Division

When the method for multiplying has been studied enough, dividing is not a problem any more. One of the lines marked by the arrows is set at the dividend, and the cursor is placed on the divisor in the slide.

Then, there is always the doubt of the scale part in which the quotient is. An analogous rule like the one used for multiplying should be used. Thus, if the dividend is set at a simple arrow (↗ or ↘), then there must be a scale between the divisor (in the slide) and the quotient (in the body). When at ⤢ or ⤡, set none or two scales.

Example: 73,4 : 8,7 = 8,435

Set the slide to the right so that the line by ↗ is under 73,4 in B, place the cursor to 8,7 in D, and then as we have used a simple arrow, there must be a scale between the numbers and the result is 8,435 found in B under the cursor.

When dividing there may be errors. For example if in the previous example it is chosen to set the slide to the left so that the line by ⤡ is under 73,4, then the cursor cannot be placed under the divisor 8,7 on the slide. With some practice, however, it is possible to tell which side to push the slide in view of the divisor.

1.4.- Combination with multiplication can of course be done without reading the intermediate result

Example: $\dfrac{17{,}37 \cdot 26{,}22}{3{,}94} = 115{,}6$

It is always advisable to start with the division in order to avoid a later slide change by means of the cursor. Thus, set the slide left so that 17,37 in C is under the line by ↘. Place the cursor to 3,94 in D. The intermediate result is now on B, but remember, do not read! Set the slide so that 26,22 in C is under the cursor, and read the result 115,6 right at the line by ↗ in C, as there is no scale between B and C.

1.5.- Reciprocal values

Set the slide so that the lines by the arrows are exactly one above the other, then the slide converts into a reciprocal table. To each number in B the reciprocal is found below in C, and vice versa. This serves, for example, to convert 1:n slopes into %.

1.6.- Powers

a) Squaring: You multiply the number by itself according to the rules under 1.1.

b) Likewise, the cube of a number can be similarly determined following the rules in 1.2.

1.7.- Square Roots

Set the slide so that the line by ↘ or ↗ point at the radicand, and then move the cursor until the same number is read in the body and the slide. This is then the square root.

Example:

a) $\sqrt{453{,}5} = 21{,}3$

Slide line by ↗ under 453,5 in B, and the result is 21,3 in A and C.

b) $\sqrt{45{,}35} = 6{,}735$

Use the same setting as in a), but now the result 6,735 is in the scales B and D.

2.- THE "PY" SCALES

These are arranged exactly as the "Lo" scales. The divided scales are hereinafter identified from below, with a and b in the body and c and d in the slide.

In order to achieve the required accuracy, the scales are provided with the unique layout having triple numbering series. The first (large numbers) ranges from 0 to 25, the second (small odd digits) from 0 to 50, the third (small decades digits) from 0 to 100. To distinguish better between the last two numbering series, it is better to mark them with different coloured ink. For orientation in the reading, auxiliary divisions are attached without numbering, to be read in relation with the scale main marks.

The selection of the number series to use depends on the greater number being added. See table below.

Number series	when the greater number is between		
0 - 25	10 - 25	or	100 - 250
0 - 50	25 - 50	or	0 - 5
0 - 100	50 - 100	or	5 - 10

But in the same operation do not add numbers with different decimal shifting, like it could be done in the "Lo" scales. If for example 250 is read instead of 25, then a zero is also to be added in reading the other numbers in the scale. The reason for this is that the "Py" scales are not periodic, like the "Lo" scales, but theoretically run to infinite without stopping at any point. Nor should different number series be used in an operation, but a single one must be used until reading the result at the number series initially selected.

2.1.- Hypotenuse from the smaller sides

Proceed exactly the same as for multiplication in the "Lo" scales. Rule: If the short sides are in a and d series or in b and c series, then read right by ↙; if they are in a and c, then read left by ↗ or ↘. In other words: if a scale is between the two scales in which the user has set the short sides, then read left to ↘ or ↗, but if none or two scales are in between, then read right at ↙.

Examples:

a) $\sqrt{21{,}94^2 + 9{,}46^2} = 23{,}89$ figure 3.
Use the 0 - 25 scale! to 21,94 in b (at ▲), set 9,46 in c (at ▼) or to 9,46 in a (at ▼) (dotted line) set 21,94 in d (in ▲), and the read the result right at ↙. Of course, this example can also be done with the series 0 - 50 or 0 - 100, but with much less accuracy.

b) $\sqrt{31{,}36^2 + 28{,}52^2} = 42{,}39$ figure 4.
Use the 0 - 50 scale! to 31,36 in a (at ▼) set 28,52 in c (at ▼) or vice versa. Read the result left by ↘.

c) $\sqrt{32{,}3^2 + 51{,}9^2} = 61{,}14$ figure 5.
Use 0 - 100 scale! To 32,3 in a (at ▼) and 51,9 to C (at ▼) or vice versa. Read the result left by ↗.

Other than these three scale combinations can never happen. If the short sides are in b and d, then the hypotenuse cannot be found on the slide. You must then use a higher number series.

If one cathetus is very small compared to the other, then it will be found that its placement in the respective scale will be very imprecise. But it can be understood that an error in the small cathetus has little effect on the value of the hypotenuse.

2.2.- Large cathetus from hypotenuse and short cathetus

Procedure as with division on the "Lo" scales.

Example: $\sqrt{23{,}89^2 - 9{,}46^2} = 21{,}94$

Set the line in the slide by ↙ to 23,89 in b scale, place the cursor at 9,46 in the c scale, and read 21,94 in b. The setting is now like in Figure 3.

Of course you can also calculate the short cathetus, but this is not recommended for known reasons, and rarely occurs.

2.3.- Expressions of the form $\sqrt{a^2 + b^2 - c^2}$, or longer like this one, can be calculated without reading intermediate results

This corresponds exactly to the calculation of the formula $\frac{a \cdot b}{c}$ in the "Lo" scales.

Example: $\sqrt{392{,}5^2 - 421{,}3^2 + 364{,}6^2} = 330{,}9$

It always starts with the subtraction, similarly to dividing, advantageously avoiding any subsequent slide change with the cursor.

Use the 0 - 50 (500) series; set slide to the right with ↘ at 392,5 in scale b, place the cursor on 421,3 in scale d. Keep in mind the scale b. Then set 364,6 in scale d under the cursor, and read the result 330,9 left by ↗, as there is one scale (c) between b and d.

Naturally, the imaginary intermediate result $\sqrt{392{,}5^2 - 421{,}3^2}$ cannot be found in the scale, it will only be marked with the cursor.

2.4.- Combined expressions of the form $\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}$ like multiplications on the "Lo" scales

When you do not know how big the final result will be, so that you cannot select a priori the correct number series, then you must be careful that during a calculation the respective result does not fall beyond the end of the selected number series. If so, then you have to read the previous intermediate result and transfer it to the next number series to continue the calculations.

Example: $\sqrt{9{,}24^2 + 10{,}9^2 + 13{,}5^2 + 15{,}14^2 + 15{,}3^2} = 29{,}15$

Use first the 0 - 25 series. When you came to include 15,14, then you have to read the intermediate result 24,81 and change to the 0 - 50 scale for the final calculation $\sqrt{24{,}81^2 + 15{,}3^2} = 29{,}15$. However, when the final result is known in advance to be larger than 25, then the 0 - 50 series is selected from the beginning and this saves reading the intermediate results.

2.5.- <u>Finally, the following calculations refer to right-angled isosceles triangles</u>

$$a = b \cdot \sqrt{2} = \sqrt{b^2 + b^2} \quad \text{and}$$
$$b = a \cdot \sqrt{2} = \sqrt{a^2 - b^2}$$

The first is calculated simply as in 2.1. The second just as the square root of the "Lo" scales. Set the slide so that the hypotenuse a is by ↗ or ↘, always to the left, never by ⤢, and move the cursor until it marks the same values in a scale (body) and c scale (slide).

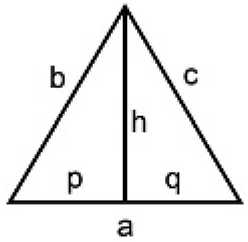
Example: $23{,}69 : \sqrt{2} = 16{,}75$
↘ to 23,69 in scale b, and the result 16,75 in scales a and c.

3.- <u>APPLICATIONS</u>

They show the added value of the slide rule to combine the use of the "Lo" and "Py" scales.

3.1.- <u>Height and base point in a triangle</u>

Formulas:



1) $m = \sqrt{a^2 - c^2 + b^2}$ $\qquad n = \sqrt{a^2 - b^2 + c^2}$
with "Py" scales like in 2.3

2) $p = \dfrac{m}{2a} \cdot m$ $\qquad q = \dfrac{n}{2a} \cdot n$
with "Lo" scales like in 1.4

Check: p + q = a, possible proportional distribution of errors!

$h = \sqrt{b^2 - p^2} = \sqrt{c^2 - q^2}$
with "Py" scales like in 2.2

Example: a = 46,23 b = 40,37 c = 37,78
m = 48,37 n = 43,99 Use 0 - 50 series!

$$\begin{array}{rl|l} p = & 25{,}30 + 1 & 25{,}31 \\ q = & \underline{20{,}91 + 1} & \underline{20{,}92} \\ & 46{,}21 + 2 & 46{,}23 = a \end{array} \qquad \begin{array}{l} h = 31{,}46 \\ \;\;\; = 31{,}42 \end{array}$$

3.2.- <u>Coordinates calculation for points (and also coordinate transformation)</u>

1) $s = \sqrt{(y_n - y_1)^2 + (x_n - x_1)^2}$ with "Py" scales

$$\left.\begin{array}{ll} \text{2)} \quad o = \dfrac{y_n - y_1}{s} & a = \dfrac{x_n - x_1}{s} \\ \text{3)} \quad \Delta y = o \cdot \Delta s & \Delta x = a \cdot \Delta s \end{array}\right\} \text{ with "Lo" scales}$$

In formula 3, the cursor remains in o or a respectively during the entire calculation.

Check: $[\Delta y] = y_n - y_1$ $\qquad [\Delta x] = x_n - x_1$

Theoretically the accumulated error must be distributed proportionally to the square of the coordinate differences. But it will always be so small that you can make the distribution without any special calculation.

3.3.- Mean error

$$\mu = \sqrt{\frac{[\upsilon^2]}{n-\nu}}$$

Calculate first $w = \sqrt{[\upsilon^2]}$ with "Py" scales like in 2.4. Then $\mu = \frac{w}{\sqrt{n-\nu}}$ with the "Lo" scales. n - ν is always an integer below 20. In order to facilitate the frequent use of the roots of these numbers, make a small table and glue it to the back of the instrument.

3.4.- Polygon and Compass surface measurements

The coordinate changes are calculated with the "Lo" scales, where the angle values may be used from a four-digit table. But it could be easier to use the "Py" scales when not so precise calculation of $s = \sqrt{\Delta y^2 + \Delta x^2}$ is required.

Other applications of the surveying practice, such as curve layout, area calculations, compensations or error distributions do not need to be particularly explained. One of the main advantages of the slide rule is that the field work can be substantially reduced by the fast and reliable control of right angles.

On the back of the instrument errors for length measurements and some commonly used formulas are given. If the specific information for railroad surveyor is not needed, then other self-written notes can be glued there. The 8 map scales on the bevelled sides will sometimes provide good service and at least will simplify the handling in the field work.

About the accuracy the following may be said: From a series of trial calculations it was found that the average linear estimation error was ± 0,063 mm, which can be applied to all parts of the scales. This means in the "Lo" scales, that the average error of the value from a calculation with 2 arguments is ± 0,4‰ of the reading, and for the "Py" scales it is from ± 0,5‰ (at the worst point: 10,0 of 0 - 25 series) to ± 0,1‰. For hypotenuses between 100,0 and 125,0 the calculation can be significantly improved if the short sides are doubled, then the result will be read in the range from 200 to 250 of 0 - 25 series, which must then be halved again. The mean error is thus down to ± 0,15 to 0,10 ‰.

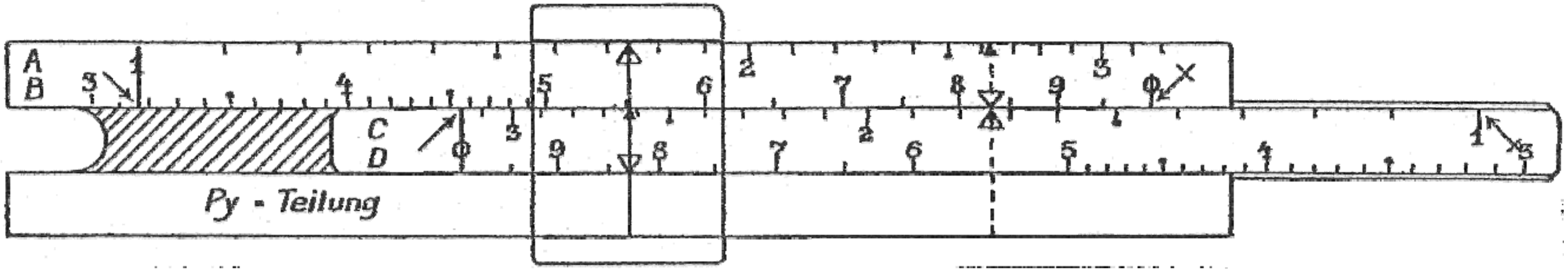
With all this, the usefulness of the instrument should be demonstrated, and its benefits may be summarized as follows:

1) Handling as with usual 25 cm slide rules.
2) Accuracy that is sufficient for the complete geodetic practice.
3) speed and reliability of the calculations.
4) Versatile availability, especially through the back tables and formulas and through the 8 map scales.
5) Easy use, with the two systems sharing common calculation rules.

Saarbrücken, February 1925,

Seifert

Figure 1



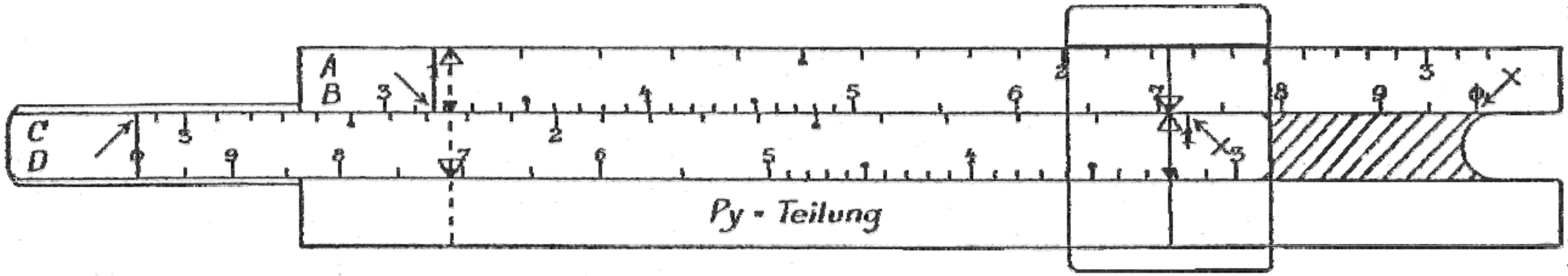


Figure 2

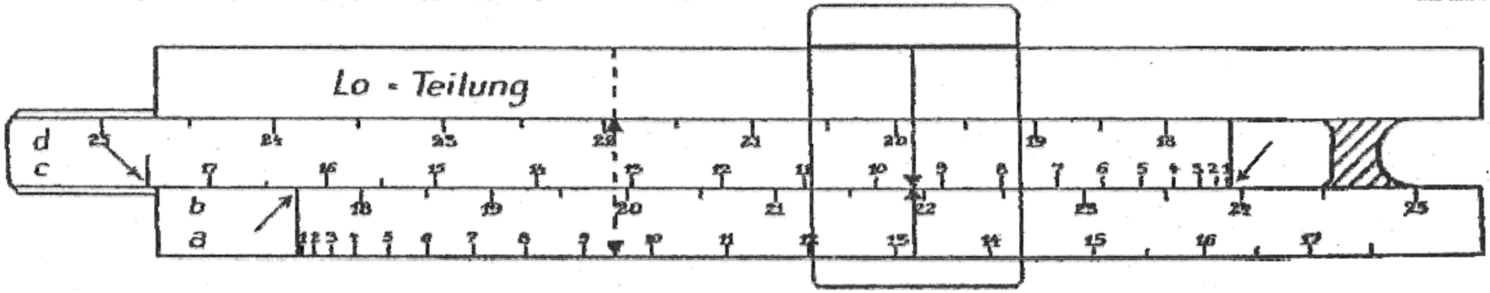
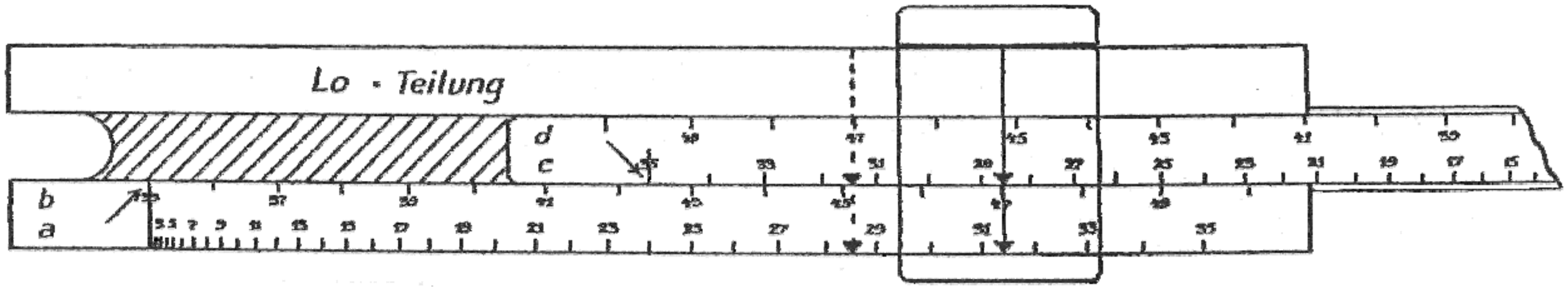


Figure 3

Figure 4



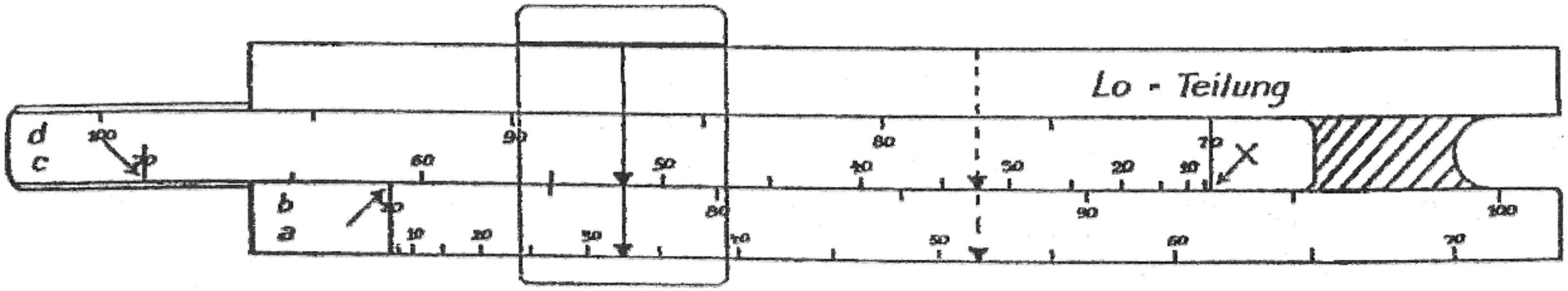


Figure 5